We put the poker in the fire

The Concept of Possibility based on Finite State Machines
Draft version

Ferenc ANDRÁS
ORCID iD: 0009-0002-9995-8173

February 16, 2025

Keywords

Metaphysics, Physical object, Necessity, Possibility, Modality, Finite State Machine, Deterministic Finite Automaton

Abstract

Deterministic finite state automata can serve as adequate models for a large class of physical objects. This is not a philosophical hypothesis but an established engineering and scientific practice familiar to all engineers and scientists. My study interprets the metaphysical concepts of possibility and necessity within the conceptual framework of automata while also addressing the question of what constitutes a physical object.

The perspective I adopt is closest to the view that a physical object is nothing more than the way it behaves in its environment—in other words; it is a system of dispositional properties along with certain essential actual properties. Since no physical object exists independently of an environment, its actual empirical properties in a given context can be understood as the realized values of its dispositional properties (e.g., an object's temperature or colour).

This is not an alternative modal logic but rather an alternative metaphysical explanation. Following scientific practice, I apply a model to explicate the philosophical meaning of the concepts of possibility and necessity.

1 Introduction

Many have suggested that the world can be described, in whole or in part, by finite automata.[1] In this case, our metaphysical beliefs about the world are not expressed directly but rather indirectly, through the conceptual framework of the automata models we employ. From the perspective of metaphysical realism, this constitutes a kind of substitute metaphysics, which is only valid if it can explain the concepts of 'possibility' and 'necessity' within the conceptual system of automata. To achieve this, it cannot rely on the metaphysical notion of 'possible worlds'; instead, it must utilize the technical notions of relations between similar 'situations' and 'alternatives' between situations—provided these notions can be traced back to the functioning of automata.

In this paper, I demonstrate how the metaphysical concepts of 'possibility' and 'necessity' can be interpreted within the framework of finite automata. My central idea is as follows: in the language of automata, possible worlds correspond to the states of automata, and the relation of alternatives between possible worlds corresponds to state transitions, as defined by the automaton's formal structure.[2] The properties of the alternative relation are determined by the types of automata under consideration. Therefore, it is necessary to classify finite automata in philosophical terms; however, before doing so, I must first clarify what a finite automaton is in my conception.

Deterministic finite automata serve as adequate models for a vast array of physical objects. This is not a philosophical hypothesis but an established engineering and scientific practice, familiar to all engineers and scientists. In this study, I interpret the alethic notions of 'possibility' and 'necessity' within the conceptual framework of automata while also assuming an answer to the question of what constitutes a physical object. My view aligns most closely with the perspective that a physical object is defined by its behaviour in its environment—in other words, that a physical object consists of a system of

[1] See Fredkin (2023) and Stephen Wolfram's foundational research on the cellular automata description of the physical world (Wolfram 2002).

[2] This line of inquiry was pursued decades ago by John McCarthy and Patrick J. Hayes, who interpreted modal concepts within the framework of classical logic. Let A be a relation on the set of situations. Then, a formula Φ is necessarily true in a situation s_1 if it is true in all its alternatives, i.e.: $\Box\Phi := \forall s_2(A(s_1, s_2) \rightarrow \Phi(s_2))$. Years later, McCarthy developed a new interpretation better suited to artificial intelligence research but less applicable to issues of metaphysical necessity. Therefore, my interpretation follows the earlier approach (McCarthy 1969).

dispositional properties alongside certain essential actual properties. Since no physical object exists in isolation from its environment, its actual empirical properties in a given environment can be understood as the realized values of its dispositional properties (e.g., an object's temperature or colour).

This is not an alternative modal logic but an alternative philosophical explanation. Following standard scientific practice, I employ a model to elucidate the philosophical significance of the concepts of possibility and necessity.

2 The Poker

The poker holds significant historical and philosophical importance. According to reports, Ludwig Wittgenstein used a poker to intimidate Karl Popper during a public lecture in 1946. The amusing incident later became the subject of a book (Edmonds 2002). More importantly, Bertrand Russell attempted to illustrate the concept of temporal change to his debate partner, McTaggart, by referring to the heating process of a poker over time. McTaggart was unable to grasp the modern mathematical-physical approach—he was not alone in this, nor would he be today—while Russell, in turn, failed to see the philosophical problem with the concept of time.

Let us now examine this issue in more detail—though we shall leave time itself untouched for the moment, letting it pass at its leisure. How does this warming process unfold? And how can it be both simply yet precisely described in mathematical terms and modelled by a finite automaton? Later, I will use this model to explicate the concept of 'possibility'.

3 Example

Suppose that the stove in which we heat a fireplace poker can be considered to have infinite heat capacity (relative to the poker) due to its continuous energy production, i.e., it maintains a constant temperature during the interaction. Then, the heating of the poker can be described by the formula:

$$T(\Delta t) = T_0 - T_1 \times e^{-k \times \Delta t} \tag{1}$$

where: T is a unary function whose value represents the current temperature of the poker, T_0 is the internal temperature of the stove—also serving as the environment of the poker when placed in the fire—T_1 is the initial temperature of the poker, k is a constant determined by the thermal, surface, and environmental heat conduction properties of the poker, Δt is the time elapsed since the start of the interaction, and e is Euler's number.[3]

The poker gradually heats up to a maximum temperature once placed in the fire.

Some effects take place immediately in an experimental setup, while others occur gradually as the environment exerts its influence over time. This difference is captured by two categories of transmission elements (or components) based on their delay characteristics: Zero-Delay Elements and Time-Delay Elements.[4]

In Zero-Delay Elements, the effect—the input signal—passes through without delay, whereas in Time-Delay Elements, the effect is delayed and only gradually takes effect. In a cybernetic framework, a poker can be considered a Time-Delay Element. The input is the ambient temperature, and the output is the poker's temperature. The phenomenon itself can be described with varying degrees of accuracy: either as an analogue signal over the range of real numbers or as a digital signal over the range of integers or their fractions. These two approaches are illustrated in the following graph of the poker's heating process (Figure 1).

Figure 1 illustrates how a continuously varying quantity can be approximated with discrete steps. To simplify the example, assume that the poker can only take on three distinct temperature states: cold, warm, or hot. Similarly, suppose that there are only two types of environments: night and day.

[3] I am grateful to physicist Pál Zimmermann for the physical explanation of the experiment.

[4] Zero-Delay Elements: These are components where the effect or output closely follows the input signal without any significant delay. In other words, the output of these elements is nearly instantaneous in response to changes in the input. Examples of zero-delay elements include resistors, capacitors, and ideal voltage sources in simple electrical circuits. In the context of signal processing, filters with zero or minimal phase distortion can also be considered as zero-delay elements.
Time-Delay Elements: Time-delay elements introduce a delay in the effect or output relative to changes in the input signal. The delay can be constant or variable, and it may be expressed in terms of time units (e.g., seconds) or phase shift (e.g., degrees). Examples of time-delay elements include inductors, transmission lines, and certain types of filters or signal processing systems that inherently introduce a phase delay. These two categories of elements are essential in various fields of engineering and physics, including electrical engineering, control systems, signal processing, and more.

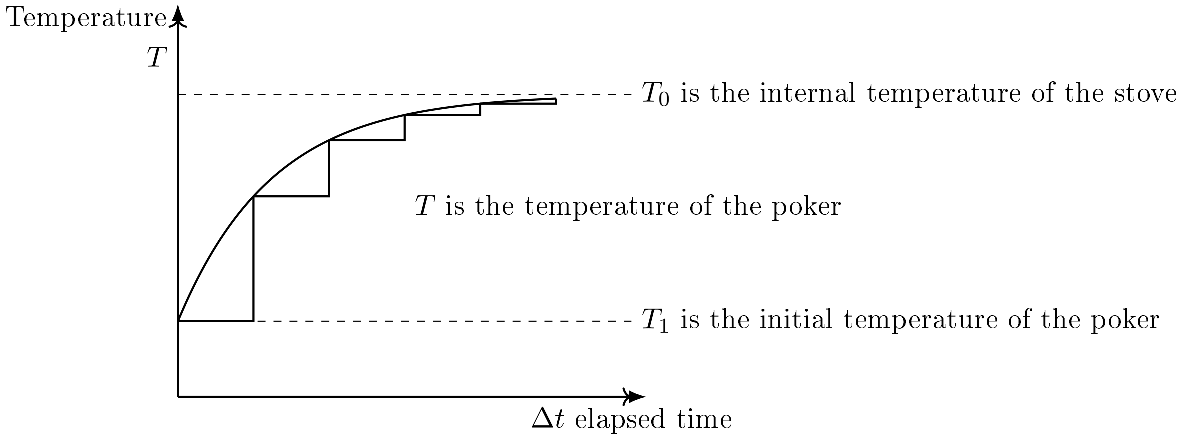

Figure 1: The heating of the poker

During the day, the poker is either black and cold or black and warm, but at night it is invisible. If it is hot, it appears red both at night and during the day.

Given a finite deterministic Mealy automaton simulating the behaviour of the poker: if the poker is placed in the fire while initially cold, it first becomes warm and then hot. If removed from the fire, it cools down gradually. The temperature of the poker represents the internal state of the automaton, while its colour represents the output state.

4 Philosophical implication of the example

The warming of the poker raises several fundamental philosophical questions:

1. As the poker heats up, its colour changes—the tip begins to glow—and as it expands, its shape alters slightly. Yet, we continue to regard it as the same object. How can we reconcile this persistence of identity with the fact that its properties differ significantly when cold and when hot? Is it possible to describe this transformation without contradiction? What constitutes the poker's identity–does something constant underlie its change?

2. Consider an empirical statement about the poker's temperature, such as "The temperature of the poker at time t_1 is 275°C." What does it

mean for such a statement to be true? How can we formally express this empirical truth within a logical framework?

3. The heating and cooling of the poker occur gradually rather than instantaneously; this is not merely a contingent fact about a particular poker but an intrinsic feature of any poker-like object. Even if the poker were never placed in the fire, we would still regard it as necessarily subject to gradual temperature change under the right conditions. This necessity suggests that the poker can be modelled as a system with a time-delay element, akin to a black box or an automaton.

4. What do we mean by 'possibility' and 'necessity' when discussing the poker's heating process? Does possibility merely signify that the poker might undergo heating or cooling at some time, or does it imply something more? Could it be possible, in some sense, that the poker never heats up at all? Is it possible something that never occur?

There exist multiple ways to heat and cool a poker, depending on the temperature of the stove, but not all possible heating curves are physically realizable. The rate of temperature change is constrained by the poker's mass and material composition. To avoid the complications associated with mathematical continuity, we approximate the heating and cooling processes using stepwise functions and consider only a finite time interval, T. In this way, the number of possible temperature histories remains large but finite, as we conceive of time as a discrete sequence of instants. This approach enables us to model the poker not only with analogue systems but also with finite-state automata operating in discrete time with a finite number of states.

5 An alternative interpretation of modality

Let Ψ denote all possible heating or cooling functions of the poker, one element of which is the φ_{reality} function describing the real history of the poker, which is only partially known up to the present moment. (See Figure 2) The function set Ψ exists because it can be determined from equation (1), since for a finite range T, under the simplifying conditions, all its elements can be computed from the given formula. (The computation can be done by a machine.) However, the function φ_0 up to the present moment–the heating or cooling history of the poker–is also an element of Ψ, because it is one of the possible functions. I denote by Ψ^{w0} the restriction of the set Ψ that contains exactly those functions that are identical to the history of the poker up to the present time, but afterwards but afterwards contain all

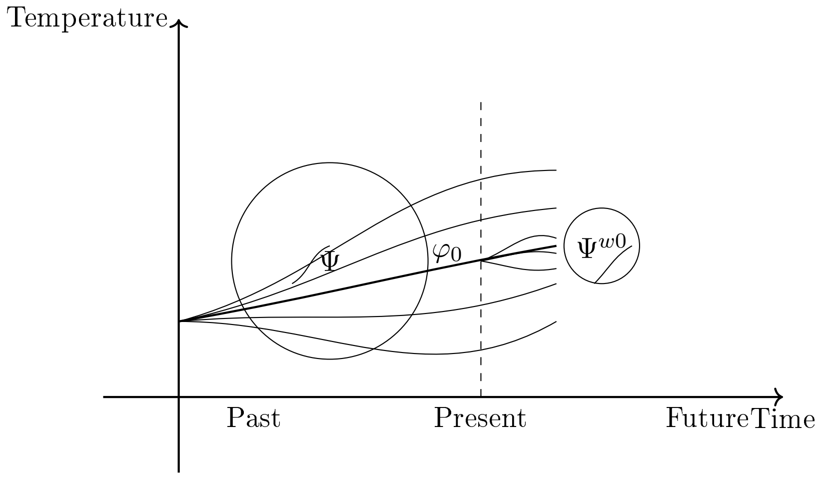

Figure 2: Possible histories of the poker

possible heating or cooling functions. φ_0 is the slice of the function φ_{reality} up to the present time. Possibility at the present time is a function φ_i that is a component of Ψ^{w0}. Both Ψ and Ψ^{w0} can be determined from the given physical formula under simplifying assumptions. To summarize concisely, we can express all this in the language of set theory, where: $t_0 :=$ present-time and w_0 is the poker state associated with t_0:

$$\Psi^{w0} \subseteq \Psi \text{ and } \varphi_0 \in \Psi^{w0} \text{ and } \varphi_{\text{reality}} \in \Psi \tag{2}$$

$$|\text{possible}(\varphi_i)|_{t_0} := \varphi_i \in \Psi^{w0} \tag{3}$$

Note well that in this conception, what is possible is what can be deduced from the heating equation of the poker, assuming some environmental effect, and what is necessarily true is the (physical) statement about the poker that is true for all possible environmental effects based on the heating equation. However, the heating equation of a poker must be formulated in some logical language in order to make a sound philosophical statement. This is done by a finite automaton model simulating the behaviour of a poker. In this way, I will derive the necessary truth of some physical statements about the poker from the properties of the poker. It is necessary that the poker heats up with a delay and cools down with a delay. To understand this, we do not need to believe in similar pokers existing in possible worlds, as David Lewis taught.

6 Finite State Machines

6.1 What is a finite automaton?

I will explain the concept of Finite State Machine (FSM) or Finite State Automaton (FSA) in ordinary language and with mathematical formulas. For philosophical reasons, I will depart from the usual terminology. In mathematics or electrical engineering, the input of an automaton can be characters of an alphabet or electrical signals, and the output can only be entities of the same type. However, the purpose of philosophical interpretation is better served by a specific terminology. In my approach, automata have input and output characteristics and internal states, described in formal language by functions. The values of these functions are states. Under certain conditions, an automaton can switch to another external (input, output) or internal state. I call an event a sequence of states, or at least two consecutive states.

A finite automaton has only finitely many different internal and external states. The current input and internal state of a deterministic finite automaton uniquely determine the current output and next internal state of the automaton at any time. This dependence is described by functions. The finite automaton operates at discrete times, which can be represented by integers. The operation of real automata that exist in the physical world is a good approximation of the above assumptions, as in electronic circuits.

Finite automata are not just black boxes. Black box theory describes the relationship between outputs and inputs in terms of functions or differential equations. For the same input signal, the black box always gives the same output signal in a consolidated steady state. The output response does not depend on anything other than the input signal. There is a functional relationship between the output signals and the input signals. Finite automata work differently. Finite automata have internal states, and the response to an input signal depends on the internal state. (Computers are essentially finite automata—their most important components can be considered finite automata. Finite automata can be complemented with infinite memory and memory read/write units to form a Turing machine, and cellular automata can be defined using finite automata.)

6.2 Mealy machines

The basic idea of Mealy's finite automata is as follows (Mealy 1955):

i. A deterministic finite state automaton always has a unique, fixed state and can only take a finite number of states. The automaton operates in discrete time. The atoms of time are sometimes called moments or beats. To reduce mathematical difficulties, we consider time to have an atomic structure in a finite domain (T). We use integers to represent the atoms of discrete time. For any time (t), the next atom of time is ($t + 1$) and the previous atom of time is ($t - 1$). Sometimes I will refer to the present time as (t_0). I will denote the next moments by $t_1, t_2, \ldots$ and the previous moments by $t_{-1}, t_{-2} \ldots$.

ii. The set of internal states is (A), the set of input states is (X), and the set of output states is (Y). The output state of the automaton is uniquely determined by the internal state and the input. In our interpretation, an automaton without output is meaningless because, without output, there is nothing to test, nothing to measure—the automaton is invisible.

iii. For a given internal state and input state, the automaton will move to the next internal state in the next cycle. All possible state transitions are described by the function:

   $\delta : A \times X \to A$
   δ : next-internal state = δ(current-internal state, current-input state).

iv. For a given internal and input state, the automaton is simultaneously set to a specific output state. All possible output state transitions are described by the function:

   $\lambda : A \times X \to Y$
   λ : current-output state = λ(current-internal state, current-input state).

v. Given all this, the finite automaton is an ordered quintuple: $M = \langle A, X, Y, \delta, \lambda \rangle$.

   (Note: In other interpretations, the definition includes the set of initial states of automata.)

7 Finite automaton model

Figure 1 shows how to approximate a continuously varying quantity with a stepwise resolution. Let's simplify the example so that there are only three different temperatures of the poker: cold, warm (lukewarm), or hot. Assume that we are only looking at two different environments, night or day. During the day, the colour is black when cold or lukewarm, but invisible at night. If it is hot, it is red both at night and during the day.

Then, given a finite deterministic automaton of Mealy's type, simulating the behaviour of a poker: if you put it in a fire and it was cold before, it will be lukewarm first and then hot. If you take it out of the fire, it cools down gradually. I take the temperature of the poker as the internal state of the automaton and its colour as its output state.

I define the internal state transition function (δ) and output state (output signal) function (λ) of the automaton using simple tables (Table 1 and Table 2). The top row contains the internal states and the left column the input states (signals). In this case, the input state is the ambient light conditions—day or night—and the distance of the poker from the fire: in fire, near, far. (In fire, the distance is zero.) The internal state is the temperature of the poker. We could work at a finer resolution, but then the table would be much more complicated and would miss the point.

δ	hot	warm	cold
in fire,at night	hot	hot	warm
near,at night	warm	warm	warm
far,at night	warm	cold	cold
in fire,daytime	hot	hot	warm
near,daytime	warm	warm	warm
far,daytime	warm	cold	cold

Table 1: Internal-state transition function

λ	hot	warm	cold
in fire,at night	red	red	invisible
near,at night	invisible	invisible	invisible
far,at night	invisible	invisible	invisible
in fire,daytime	red	red	black
near,daytime	black	black	black
far,daytime	black	black	black

Table 2: Output-state function

Note the approach of the model. The elements of the sets A, X, and Y are possible states, not actual states. Functions do not describe actual state changes or transitions, but possible transitions. The actual states and transitions can only be represented in cyberspace by the temporally existing, working version of the model, or can be computed from the tables if the input states are known. In the world of static texts, the model does not change, it is not alive, it does not respond to interactions. Instead, it defines all possible transitions for each possible input state (signal). This is the philosophical basis of modality simulation.

This is not at all unusual, since all physical laws operate with relations of possible values.

In Newtonian physics, any two bodies are mutually attracted. Between two bodies that can be considered point-like, this force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. The law does not apply to particular physical objects or celestial bodies—within its limiting conditions—but to any (possible) object of mass, and its truth is not contingent but necessary. The formula F = m × a, familiar from high school, does not describe a specific event, nor does it define the relationship between specific forces, masses, and accelerations in classical physics, but rather the relationship between possible physical values. The modal concepts are there in physics, the possibilities are there in reality, because the laws of nature are the basis for the possibilities; they do not need to be imported from outside.[5]

8 Functions and sentences

In general, the set of functions is larger than the set of propositions. However, in our investigation, we are only concerned with functions that have a finite domain and a finite range. This is why we previously provided tables of functions describing automata. These tables allow us to describe the states of finite automata in a formal logical language. The output function and the internal state transition function can be expressed as follows:

$$y(t) = \lambda(a(t), x(t)) \tag{4}$$

[5]Similarly, Samuel Kimpton-Nye investigates the common ground of modal concepts and natural laws. (Kimpton-Nye 2018)

(5) $$a(t+1) = \delta((a(t), x(t))$$

The operation of a simple finite automaton simulating the behaviour of a poker can be represented using tables or formula sets. Both representations are equivalent. Below, we provide a table-based description followed by its equivalent formalization.

First row:

1.1. If the poker is in fire at night and is hot, it remains hot, and the tip of the bulb is visible.
If$\langle$in fire, at night$\rangle = x(t)$and hot $= \delta(t)$then red $= y(t)$and hot $= \delta(t+1)$
1.2. If the poker is in fire at night and is still warm, it becomes hot and the tip of the bulb is visible.
If$\langle$in fire, at night$\rangle = x(t)$and warm $= \delta(t)$then red $= y(t)$and hot $= \delta(t+1)$
1.3. If the poker is in fire at night and is still cold, it will be warm and will not be visible.
If$\langle$in fire, at night$\rangle = x(t)$and cold $= \delta(t)$then invisible $= y(t)$and warm $= \delta(t+1)$

Second row:

2.1. If the poker is close the stove at night and hot, it cools down and becomes warm and invisible.
If$\langle$close, at night$\rangle = x(t)$and hot $= \delta(t)$then invisible $= y(t)$and warm $= \delta(t+1)$
2.2. If the poker is close the stove at night and warm, it remains warm and invisible.
If$\langle$is close, at night$\rangle = x(t)$and warm $= \delta(t)$then invisible $= y(t)$and warm $= \delta(t+1)$
2.3. If the poker is close the stove at night and cold, it becomes warm and remains invisible.
If$\langle$near, at night$\rangle = x(t)$and cold $= \delta(t)$then invisible $= y(t)$and warm $= \delta(t+1)$
etc.

The two descriptions (Table 1.,2. and 1.1 . . . 2.3 formulas) are equivalent. The description of the automaton $M = \langle A, X, Y, \delta, \lambda\rangle$ by functions or tables in formal language L is denoted by the set of sentences $L_{\langle A,X,Y,\delta,\lambda\rangle}$, and the description of the state of the automaton s in formal language L is denoted by $\rho(s)$.
This formalization ensures that the description of the automaton is fully captured in logical terms, allowing for analysis using formal methods.

9 Types of automata

I consider both physical entities–such as electronic circuits–and abstract mathematical structures as finite automata. If it is not clear from the context, and the distinction is essential, I will indicate separately which one is meant. The operation of a physical automaton can be classified in several ways and can be considered as an implementation (model) of different abstract automata. "Whether an automaton found in practice is deterministic or stochastic does not really depend on the automaton itself; rather, it depends on the model used, the viewpoint and the degree of accuracy that describes the operation of the automaton."(Adam 1972) In the following paragraphs I will consider automata that are assumed to operate completely deterministically after the initial state, and only produce a finite number of different input and output states.

The number of external characteristics An automaton can have more than one input or output, but for simplicity I will often simply refer to an 'input' or 'output' instead of a sequence of inputs or outputs at the same time. If necessary, I denote the i-th input by x_i and the j-th output by y_j.

Start-useful automaton It is assumed that the automaton can reach all possible internal and output states at least once for some input state sequence. In short, the sets A and Y have no redundant elements. This type of automaton is called a 'start-useful' automaton (Daciuk 1998).

Generators, clocks If X–the set of input states–is a one-element set, then I call the automaton a "generator". In this case, the set X can be omitted, because the behaviour of the automaton is fully described by the functions $\delta = A \to A$ and $\lambda = A \to Y$. An automaton considered as a generator produces a constant output state or a sequence of output states without any input events. Generators are independent of their environment.

Combinatorial systems A combinatorial automaton is an automaton in which any input state always produces the same output state, regardless of the internal states. In this case, there is only one element of the internal states of the set A, and it can be omitted. This means that the combinatorial automaton can always be described without taking into account the internal states, but only by a function between the input and output states. Black boxes can be considered as combinatorial automaton.

Steady state combinatorial automaton Some machines can be considered as combinatorial machines in an extended sense. The operation of these automata is only briefly influenced by their internal states, but after a while, the output state of the automaton is clearly determined by the input state. So, in the steady state, the operation of these automata can be described as similar to a combinatorial automaton, where there is a function relation between input and output. I call a 'steady state combinatorial automaton' an automaton which, after a transition time $0 < \Delta$, always behaves like a combinatorial automaton, and then its output is determined by its input, independently of its internal state. (The heating of a poker is such an automaton.) Automata that never reach a steady state are astable systems, those that reach a steady state after a finite time are stable systems.

Sequence systems (circuits) "If the current inputs are sufficient to determine the outputs, then the system is a combination system. If the control system needs additional information about the sequence of input changes to determine the outputs, then the system is a sequential system."(Wagner 2004) Most finite automata are sequential systems. Sequential systems (automata) are sometimes called "memory-based circuits" because they respond to input states as a function of previous events.

Cyclical, acyclical, ccyclical and tcyclical automata If it is possible to exit from any state and to return to the original state after one or more transitions as many times as necessary, the automaton is cyclic; on the other hand, if all the internal states of the automaton can only be exited but not returned to, or only entered but not exited from, the automaton is acyclic. Acyclic automata do not contain cycles, i.e. they cannot return to the same state twice after transitions. There are different types of non-cyclic automata. Conditional cyclic (ccyclical) automata can return to the same internal state at any time, as long as they do not switch to a specific internal state. Transient cyclic (tcyclic) automata are able to return to the same internal state for a period of time, but after a given $n \geq 1$ cycle they start to behave like acyclic automata. An automaton can be both a conditional and a transient cyclic system, but a cyclic automaton cannot be either a ccyclic or a tcyclic automaton.

From a physics point of view, cyclic automata are reversible systems, while non-cyclic automata are irreversible physical objects. Some sequential automata have a cyclic structure, others do not, but all combinations of automata are cyclic, and all non-cyclic automata have a sequential structure. This division is similar to analogue systems:

Finite Automaton	Analogue System
Generator	Oscillator
Combinatorial System	Ideal linear system
Steady state combinatorial automaton	Linear filter
Sequence System	Non-linear system

Table 3: Analogy Between Finite Automata and Analogue Systems

10 Philosophical interpretation

An automaton can serve as a model for formal or formalized theories as well as for physical entities. Everything that can be described at the level of propositional logic can be modelled by finite automata, as truth functions can be easily simulated by combinatorial automata (digital circuits). However, in this paper, I will focus solely on the automaton model of concrete (physical) objects.

Following recent research trends in physics, some physical theories suggest that fundamental physical properties such as mass, force, temperature, charge, and especially space and time, have finite, discrete values. Consequently, deterministic or probabilistic finite automata can serve as adequate models for most physical objects. Some objects may suffer irreparable damage or be destroyed entirely, existing only once; accordingly, they can only be modelled as non-cyclic automata.

A plate spring always returns to its original position unless it is permanently deformed or breaks due to excessive load. If it fails after a finite number of deflections ($1 \leq n$), plate springs can thus be considered conditional and transient cyclic automata. Living organisms exhibit similar characteristics in terms of life conditions and lifespan. Upon closer examination, the behaviour of most objects resembles that of sequential automata, as they respond to external influences based on previous input events. However, the open question remains whether all material objects can be considered "start-useful" automata—namely, whether any state of the object can be induced in response to some environmental influence. This question parallels Barbara

Vetter's exploration within a metaphysical framework: can certain physical objects possess potential properties that never manifest, never emerge, and never become actualized?

Let M be a finite automaton modelling the behaviour of a material object or a living being. The object and its corresponding automaton model M always exist within a specific environment and internal state. The object's behaviour in a given environment is simulated by the machine's output states in response to its input states. This implies that the object's behaviour in its environment mirrors the automaton's behaviour within its own operational framework. The description of the automaton specifies, for each internal state, which input state produces which output state. If we map the input states of the model back to the object's environment and the machine's output states back to the object's properties, we can predict how the object will react to various situations. In this way, we can assess the applicability and accuracy of the model.

11 Relationship with logic

The fact that an object o has property F at time t_1 can be easily described by a simple formula $F(o, t_1)$. In first-order logic, the evaluation of the formula $F(o, t_1)$ determines its truth value. In the world of automaton models, the formula $F(o, t_1)$ can be represented by an automaton that has a specific output state at time t_1. Accordingly, the automaton associated with the object has a specific output state if the formula is true and another output state if the formula is false.

This changes when we consider the possibility that an object o has property F at time t_1. Translated into the language of automata, this means that it is possible for the output of an automaton to be in a particular state at time t_1. The automaton cannot switch to any internal state arbitrarily based on external influences; rather, it can transition only to specific internal states. Thus, the state it enters depends on both its previous internal state and the external influence it experiences. The regular operation of the automaton prescribes its possible output states.

In the world of automata, a possible state is one that can occur as a result of some transition, whereas an impossible state is one that can never be reached. The properties of an automaton determine which transitions are available from each state—that is, which transitions are possible and which are not. The possible worlds in the domain of automata correspond to the states of automata, and the alternative relation between possible worlds in modal logic aligns with the transition relation between states of automata. This transition relation possesses formal properties similar to the alternative relation in modal logic.

The essence of my argument is that the formal properties of the alternative relation (reflexivity, transitivity, symmetry) are ad hoc prescriptions in modal logic but emerge naturally from the types of automata in the world of automata. Returning to the real world from the realm of automaton models, this implies that physical objects—modelled by automata—determine the formal properties of alternatives in reality. Depending on whether we consider physical objects as reversible or irreversible systems, and accordingly, which type of automaton we use to simulate their behaviour, we obtain different alternative relations.

12 Definition of Possible Worlds and Alternative Relation

An automaton $M = \langle A, X, Y, \delta, \lambda \rangle$ has only finitely many different states defined by the functions δ and λ. The input, internal, and output states associated with the same time are denoted by w_i labels. A finite set of these labels: $W = \{w_0, w_1, w_2, \ldots w_i \ldots\}$. The current state of the automaton is $\langle t_0, w_0 \rangle$. $\rho(t_i, w_i)$ represents the state description at time t_i, where w_i is a label and t_i is a time. The present state of an automaton is $\langle t_0, w_0 \rangle$. $\rho(t_i, w_i)$ describes the state at time t_i, where w_i is a label and t_i is a time. The expression $\rho(t_i, w_i)$ represents a sentence that specifies the input, internal, and output states of the M automaton at time t_i.
The definition of M automata defines two types of binary relations. The definitions are:

(6) $$\begin{gathered} A_M(w_i, w_j) := \text{ a change in input characteristic } x_i \text{ to } x_j \text{ causes } M \\ \text{automata to move from } w_j = \langle x_i, a_i, y_i \rangle \text{ to } w_j = \langle x_j, a_j, y_j \rangle \text{ where:} \\ x_i, x_j \in X,\ a_i, a_j \in A,\ y_i, y_j \in Y,\ a_j = \delta(x_i, a_i),\ y_j = \lambda(x_j, a_j) \end{gathered}$$

(7) $$R_M(\langle t_1, w_1 \rangle, \langle t_2, w_2 \rangle) := t_2 = t_1 + 1 \text{ and } A_M(w_1, w_2)$$

In ordinary language: $R_M(\langle t_1, w_1 \rangle, \langle t_2, w_2 \rangle)$ holds exactly if, starting from the state w_1 of the automaton at t_1, there is an input state such that

the automaton is in the state w_2 at the next time t_2.
Call the pairs $\langle$moment, state name$\rangle$ 'possible world' or 'possible state' or 'situation', and say that $\langle t_j, w_j\rangle$ is an alternative to $\langle t_i, w_i\rangle$ possible world exactly if $R_M(\langle t_i, w_i\rangle, \langle t_j, w_j\rangle)$.

Let us consider a finite automaton whose mathematical model is M. Consider all the temporal sequences of possible worlds belonging to M, where the temporally consecutive members of the sequence are alternatives of each other. Thus, if $\langle t_1, w_1\rangle$ is followed by $\langle t_2, w_2\rangle$, then $R_M(\langle t_1, w_1\rangle, \langle t_2, w_2\rangle)$. These sequences describe the possible histories of the automaton.

The mathematical properties of the R_M alternative relation (symmetric, asymmetric, etc.) depend on the types of automata. (The study of this relation is beyond the scope of this paper and is the subject of further investigation.)

Since we assumed earlier that T—the set of time instants—is finite, the number of possible worlds (possible states) and possible histories (possible sequences) is finite. The complete set of sequences contains all possible state changes of the automaton, i.e., its transition from one possible state to another within the time interval T. I denote the set of all possible state changes (histories) of the automaton by Ψ. Thus, Ψ contains all possible state transitions, all possible histories of the automaton. Among the possible worlds, there will be one and only one φ_{reality} series such that every $\langle t_i, w_i\rangle$ situation is a member of φ_{reality} series exactly if the automaton is in state w_i at time t_i, in formal language: $\rho(t_i, w_i)$. The φ_{reality} series thus contains the real history of the physically existing automaton M, and obviously φ_{reality} in Ψ. The φ_{reality} series can be decomposed into two parts. The first part is from the initial state of the automaton to its present state—denote this by φ_0—the second part of the series contains the future states of the automaton. (Future states are not known except for generators without input; present or older states may be known partially or completely.) Consider a restriction of set to Ψ^{w0} that contains all and only those histories of the automaton that are identical to the actual history of automaton M up to the present time, i.e., φ_0. Clearly, $\Psi^{w0} \subseteq \Psi$, $\varphi_0, \varphi_{\text{reality}} \in \Psi^{w0}$. (See Figure 2) The description of the $s \in \Psi^{w0}$ or $s \in \Psi$ states, which form the elements of the Ψ^{w0} and Ψ sets, is the sentence $\rho(s)$. We now have the concepts we need to define the concepts of possibility and necessity in the world of automata.

13 Possibility and Necessity in the World of Finite Automata

Since finite automata have only finitely many states, and we are considering their operation over a finite T domain of atomic structure time, quantifiers can be considered as multiple conjunctions or alternations, and consequently these automata can be described in the language of propositional logic. Let L be the language of propositional logic describing finite automata. The atomic sentences of language L are state descriptions of finite automata M, and their molecular sentences are sentences formed by logical connectives from the state descriptions considered as atomic. Propositional logic languages are negation-complete, i.e., they can be given a set G of atomic propositions such that any of their formulas, or negations of the formulas, can be derived from the set G. (Shoenfield 1967, Bell 1977, Kleene 2002) Based on this, a sentence named x in the language L is true if and only if the sentence named x can be derived from G.

Let $L_{\langle A,X,Y,\delta,\lambda\rangle}$ be the functional description of the automaton in L formal language. Let $\rho(s)$ be an atomic or molecular sentence in L for the automaton $M = \langle A, X, Y, \delta, \lambda\rangle$, and let $\rho(\varphi_{\text{reality}})$ be the sentence describing the actual history of the automaton in L. Define Ψ^{w0} as the set of all possible histories of the automaton up to the present time, which coincides with the actual history of the automaton. We then introduce the following definitions, treating modal expressions as meta-linguistic predicates with sentence names as arguments:

Possibility in the Present:

$$|\Diamond\ulcorner p\urcorner|_{w0} := \exists s(s \in \Psi^{w0} \& (L_{\langle A,X,Y,\delta,\lambda\rangle} \cup \{\rho(s)\} \vdash p)) \quad \text{where ` } \vdash' \text{ a sign of logical deducibility.} \tag{8}$$

(A proposition p about some automaton M is possibly true in the present if and only if it can be derived from the definition of the automaton by some proposition $\rho(s)$, where $s \in \Psi^{w0}$).

Truth in Reality:

$$\text{True} - \ulcorner p\urcorner := (L_{\langle A,X,Y,\delta,\lambda\rangle} \cup \rho(\varphi_{\text{reality}}) \vdash p) \tag{9}$$

(A proposition p about some automaton M is true in the story φ_{reality} if and only if it can be deduced from the definition of the automaton and the proposition $\rho(\varphi_{\text{reality}})$).

Necessity in the Present:

$$|\Box\ulcorner p\urcorner|_{t0} := \forall s(s \in \Psi^{w0} \rightarrow (L_{\langle A,X,Y,\delta,\lambda\rangle} \cup \{\rho(s)\} \vdash p)) \tag{10}$$

(A proposition p for some automaton M is necessarily true in the present if it can be derived from the definition of the automaton by any proposition $\rho(s)$, where $s \in \Psi^{w0}$).

Note the above definitions of modal concepts. The formal symbols $(\Box, \Diamond)$ for the concepts 'necessary' and 'possible' occur on the left side of the definitions. On the right side, only set-theoretic-logical symbols and automata-theoretic concepts are used in the definitions. The above two (8,10) modality definitions use modal words in a de dicto sense; the de re interpretation is subject to further investigation.

It can be seen that, according to the above definitions, only a future event can be contingent, while the present and the past are necessary. This is because, for the operation of the automaton, the past and present are immutable; only the future is open. In turn, any present or past state of the automaton can be contingent or necessary from an even earlier state, depending on how the automaton operates. So in the world of finite automata:

(11) Everything that is past is possible because it has happened,
and necessary because it has happened and cannot be changed.
Since the present has happened and, like the past, cannot be changed,
the present is necessary.

(12) A proposition describing the future is possibly true if, starting
from the present, the automaton has an alternative set of circumstances
that makes it true.

(13) A sentence describing the future is necessarily true if, starting
from the present, all alternatives of circumstances make it true.

14 Refutation of the Master Argument

Diodorus Cronus argued that nothing is possible unless it is true or will be true at some point. However, I contend that (11) and (12) demonstrate, on the contrary, that the following three statements are satisfiable within the framework theory and thus do not entail a contradiction:[6]

(A) Every past truth is necessary.

(B) The impossible does not follow from the possible.

(C) There is a possible truth that is neither true nor will be true

To establish the satisfiability of these three statements, let us examine a model.

Suppose I purchased the poker at time t_{-5} when it was cold and black. To date, I have only placed it in the fire twice, meaning it has never been truly hot—only warm at t_{-4} and t_{-2}. At the present moment, t_0, it is warm, having been briefly exposed to fire. However, assuming I never use it again, it will remain cold. Can the previously introduced poker automata model verify that it is still possible for the poker to become hot at some future point?

The φ_{reality} series is represented in the following table (disregarding the poker's colour for now): φ_{reality} series is shown in the table below. (I have omitted the colours of the poker.)

Table 4: φ_{reality}

Temperature	cold	warm	cold	warm	cold	warm	cold	cold	cold	cold
Position	far	near	far	near	far	near	far	far	far	far
Time	t_{-5}	t_{-4}	t_{-3}	t_{-2}	t_{-1}	t_0	t_1	t_2	t_3	t_4

The sequence φ_0 consists of:

Table 5: φ_0

Temperature	cold	warm	cold	warm	cold	warm
Position	far	near	far	near	far	near
Time	t_{-5}	t_{-4}	t_{-3}	t_{-2}	t_{-1}	t_0

Now, consider an alternative sequence φ_1:

(A) From the model above, it follows that $\varphi_1 \in \Psi$. Since φ_0 is an initial segment of φ_1, it follows that $\varphi_1 \in \Psi^{w0}$. Consider the proposition

[6] Cf. Pascal 2016.

Table 6: φ_1

Temperature	cold	warm	cold	warm	cold	warm	cold	warm	hot	warm
Position	far	near	far	near	far	near	far	on fire	on fire	far
Time	t_{-5}	t_{-4}	t_{-3}	t_{-2}	t_{-1}	t_0	t_1	t_2	t_3	t_4

that the poker was cold at t_{-1}. This is a past truth, which follows from φ_{reality} and, being true in φ_0, it is true in all elements of Ψ^{w0}. Therefore, the statement "the poker was cold at t_{-1}" is necessarily true. The same reasoning applies to other past and present truths, thereby proving (A).

(B) Suppose q is possible, and q follows from p, but p is impossible. If p is impossible, then there is no $\varphi_i \in \Psi^{w0}$ such that p can be derived from it using the automaton model, since the model is consistent. However, p is derivable from q. If no $\varphi_i \in \Psi^{w0}$ allows the derivation of p, then no $\varphi_j \in \Psi^{w0}$ allows the derivation of q, which means q is also impossible. But this contradicts the assumption that q is possible, forcing us to reject the assumption. Since p and q were arbitrary propositions, this establishes (B).

(C) Now consider the proposition that the poker is hot at t_3. This is false in φ_{reality} but true in φ_1. Therefore, an element of Ψ^{w0} demonstrates that the poker can be hot at t_3 even though it is not actually hot at t_3 in φ_{reality}. The same reasoning extends to other future statements in Ψ^{w0}. Thus, (C) is established.

Note that necessity in these cases is relative to the present. Moving the present backward, the sentence "the poker was hot at t_{-2}" transitions from necessary to possible, while shifting it forward causes "the poker is hot at t_3" to move from possible to impossible.

15 Outlook

Nathan Wildman writes:

> In many ways, dispositionalism is an attractive approach to modality. For one, it is ideologically parsimonious: it has only one primitive–dispositionality–to which everything else reduces (or can at least be defined in terms of). For another, it promises

> an account of metaphysical modality in terms of actual, concrete objects and their properties. Consequently, dispositionalists do not need to postulate non-actual entities (e.g. the various denizens of Lewisian possible worlds) to serve as the anchors for modality. Finally, and relatedly, by anchoring modality in the dispositions of ordinary, actual objects, dispositionalism offers an extremely plausible epistemology of modality. ... This is a particularly appealing result because many of the competing accounts of modality–e.g. Lewisian realism and Finean essentialism–make the epistemology of modality extremely mysterious. For these (and other) reasons, a number of philosophers have recently begun developing versions of dispositionalism.

The researches of Ferenc Huoranszki (Huoranszki 2022) and Barbara Vetter (Vetter 2015) also fit into this line. According to Jennifer McKitrick, Barbara Vetter's book stands out among recent books on dispositions, so we will only briefly discuss her theory here.
"According to Vetter, saying that something has a disposition, like fragility or flammability, is to say something about what it can do, such as break or burn. Dispositional concepts are members of a broader class of modal concepts, which also includes necessity, possibility, causation, laws, and essence. Vetter's basic idea is that potentialities are fundamental, and other modal notions should be understood in terms of them." (McKitrick 2019)

Vetter argues that it is a hopeless enterprise to reduce modal concepts to purely non-modal concepts. A more fruitful approach is to select an irreducible modal concept, which is taken as a primitive basic concept, and to derive the other modal concepts from it. Vetter distinguishes potentiality from possibility, and bases the latter on the former. Central to his idea is the insight that potentiality (ability) has degrees and measures. For example, a physical object can be fragile to varying degrees, and is only considered fragile if this degree exceeds a threshold. Thus, "x is more fragile than y just in case the proportion of worlds where x has its relevant intrinsic features and breaks is greater than the proportion of worlds in which y has its relevant intrinsic features and breaks." (Vetter 2015:78)

Note that this is not how engineers and physicists understand fragility. They define a measurement procedure to measure fragility, and assign numbers to the fragility based on the measurement, with the higher (or lower) value of the number indicating greater fragility. According to Vetter's theory, there are many different varieties of potentiality: intrinsic potentiality, joint potentiality (e.g. playing the piano four-handed), extrinsic potentiality (the ability to perform a certain aerobics stunt), iterated potentiality (the ability to cultivate a friendship with someone who can play bridge). She does not lose sight of the problem of the never realised, never manifested intrinsic potential.

Vetter challenges the usual conditional analysis of dispositions. In the usual interpretation, for example, if x is fragile, then (if x had been dropped, x would have broken.) She says that this interpretation is therefore not good, because if a fragile object falls on a soft surface, it will not break, and an unbreakable glass will break if you hit it with a sledgehammer. Following this, the basic idea of the central modal concepts in Vetter's framework theory goes like this:

> Possibility*: It is possible that $p :=$ Something has, had, or will have a potentiality for it to be the case that p. (Vetter 2015:194)
>
> It is necessary that $p :=$ just in case nothing has, or had, or will have a potentiality to be such that not-p. (Vetter 2015:203)

Jessica Leech points out that Vetter's theory of modality, which traces modality back to the notion of potentiality, is similar to Kit Fine's essentialist view that things have an essential nature and that it is metaphysically necessary that p precisely if, given the essential nature of things, it is true that p. Both views ground non-local metaphysical modality in a local, particularized conception of modality. But while the essentialist starts from the essentiality (essences) that ground necessities, Vetter starts from the potentialities that ground potentialities (Leech 2017).

Vetter's understanding:

> x - soluble $:=$ x - has the physico-chemical property of being able to dissolve

It is worth comparing this with Carnap's classic approach to the reduction axiom (Carnap 1936):

> x - soluble $\rightarrow$ (if x is placed in water in the T-time domain, then if x is soluble, then x will dissolve in the T-time domain)

> x - insoluble $\rightarrow$ (if x is put into water in the T-time domain, then if x is insoluble, then x will not dissolve in the T-time domain)

Carnap's conception is closer to the way of thinking in engineering and the natural sciences, because they (predominantly) use extensional logic, although the domain of interpretation of the variables in the formulas is a set of possible values, not a set of actual values.

Vetter formulates the truth conditions of the modal term 'could' as follows:

> (Could): 'If x were F, then x could/might be G' is true iff x has an iterated potentiality to be G, and being F is an earlier stage in that iterated potentiality. (Vetter 2015: 226).

However, this differs from the meaning of 'would' in deeper analysis. According to Giulia Casini, we can summarize the difference between the meaning of might/could and would (in the dynamic=alethical sense of the term): the former indicates that the consequence is possible given the existence of the antecedent, while the latter indicates that the consequence is necessary given the existence of the antecedent. The relationship between the two concepts is, in Casini's view, too simplistic in Vetter's conception and needs further elaboration. (Casini 2022)

At the same time, the dispositionalist interpretation of modality faces a serious difficulty. How can we trace back "It might rain tomorrow." to a dispositional statement? Vetter answers the question by talking about potentiality instead of dispositions.
Potentialities differ from dispositions in several ways. Potentialities are not context-dependent, and while standard dispositional descriptions typically suggest that the probability of a response to a condition is very high, the probability of a potentiality manifesting itself ranges from unlikely to necessary. Potentialities thus range widely.
Vetter sees potentialities as properties and interprets properties as realities. In this spirit, a property is fully instantiated (fully present) in a physical object that possesses the property. Potentialities are properties, so they are also fully instantiated in objects. Suppose I have a very fragile champagne glass and a less fragile tea cup. The fragility of the two objects as two different potential properties is different. But then, how can these two different properties, i.e. the two different fragilities, be the same property, i.e. both manifestations of fragility? One possible answer to this problem assumes

a hierarchy of universals, where higher level universals contain lower level universals.

> Higher-order determinable universals are not instantiated by particulars directly by being present in them, but indirectly, in virtue of their lower-order determinate universals being instantiated. Since this hierarchical account posits universals that are not present in any particulars, it is friendlier to transcendent realism, according to which universals can exist without being present in objects. But such a Platonic view is in tension with Vetter's motivation to ultimately ground modality in the ways that actual, concrete objects are.(Mckitrick 2018)

Vetter's metaphysical theory is more widely applicable than the cybernetic conception presented here, but neither theory can cope with the problem of possibly existing or non-existing objects, which Lewis's realism can. This is a major shortcoming, because most philosophers believe that the existence of all physical objects is contingent, and that a philosophical explanation must somehow account for this.

16 Closing words

Philosophical speculations about the possibility of the number of planets or the necessary identity of the Evening Star and the Morning Star are less fortunate, because we know little about how the history of the Solar System would have unfolded if the Great Programmer had set the initial conditions differently. Moreover, what we do know cannot be well modelled by simple mathematical-logical tools. In contrast, the laws of nature governing medium-sized physical objects, machines, or living things lend themselves much better to such modelling. That it is impossible for a poker to remain cold for a long time when left in a fire is a much more defensible position than the impossibility of there being two planets in the sky instead of the planet Venus. Aristotle was wise to focus on such things, even if he was unaware that they were not typical entities of the universe.

From the standpoint of philosophy, the theory of automata allows a sufficiently general and formally precise formulation of the concept of philosophical necessity. This is because the logical structure of the laws of nature is central to philosophical inquiry. The automata model defines relations rather than considering an object as a set of ad hoc properties. Nor do we have to decide what the essential and contingent properties of an object are; rather, the model determines what the object is by recording how it behaves in its environment. Knowing the model, we can then reduce the notions of possibility and necessity to relations between the properties of objects, assuming automata serve as models of living or physical entities.

The truthmakers of modal concepts are functions of the chosen ontology. However, ontologies are dependent on framework theories, often shaped by the language we use, what we seek, and what we find in the world. Whether we populate the world with possible situations, counterfactual conditionals, or natural laws, each choice carries advantages and disadvantages. The selection of a grounding framework for modal concepts is thus shaped by one's broader metaphysical commitments. The automata model-based framework theory of modality presented here commits us to a belief in natural laws as Platonic universals in addition to physical objects. The instantiations of natural laws are simulated by automata describing the behaviour of objects.

From a logical point of view, the conception of necessity presented here is an interpretation and further development of Rudolf Carnap's conception (Carnap 1936). This interpretation holds that a proposition p is necessary if it can be deduced from the rules governing automata. Functions describing the operation of automata specify what the output state of the automaton would be given a particular input state. Thus, these functions effectively describe counterfactual dependencies. Consequently, an approach based on dispositions aligns with a framework grounded in the operation of automata.

In my conception, necessity is bound to a framework and is not an operator but a meta-linguistic predicate, making it non-trivially iterable (Halbach 2009). At present, I do not see a way to transform my account to remove this limitation. However, there is no need to assume possible worlds outside the objects described by the models; instead, possible worlds are reduced to a set of internal and external states of automata as possible states. It is, however, necessary to assume the existence of natural laws beyond individual physical objects, which are described and modelled by mathematical formulas capturing physical properties and relations between possible physical states. This approach provides a coherent interpretation of two key ideas:

1. The relational nature of properties – Properties exist in relation to models as their possible states. Philosophically, this implies that phys-

ical properties exist in relation to objects as instances of natural law, yet natural laws themselves are independent entities, not reducible to mere sets of instances. This Platonist conception is reflected in the construction of object-oriented programming languages and databases: in a programming language, properties can exist without instances, just as a database may define fields that lack values for any record.

2. The full scope of properties – The scope of properties extends beyond currently instantiated values to all possible values. This follows from the fact that the operational contexts determining behavior define transitions between all possible states, not merely the states currently realized. Thus, they encompass the full set of possible states, and each state represents a mapping of the dispositional properties of the physical object.

If we consider physical objects as systems of empirical properties, then in this view, an object is identical to the model that simulates it. This provides a response to the question of the identity of physical objects: while an object undergoes change, its internal coherence in response to environmental influences remains intact, and its operational rules predict its actual properties under varying conditions.

The working version of the models can be downloaded from the internet:

- Poker model:`https://sht.andrasek.hu/poker-en.xlsx`

- Wire model: `https://sht.andrasek.hu/wire-model.xlsx`

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