The New Clothes of Paradox [1]

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Introduction

At first glance, the most significant element in debates about Yablo's paradox has been the problem of circularity. However, on closer investigation a second level of the controversy emerges – that is, the selection of the formally correct language in which the sequence can be formulated and, following deductive inference, the contradiction derived. In fact, this latter element is the most important: the existence of Yablo's endless sequence. What kind of entities are we talking about – a list of formulae without interpretation, or a sequence of statements, where statements are the truth-bearers?

The approach adopted in this paper to the formulation of Yablo's list is, to some extent, similar to Thomas Forster's solution. However, he claimed: "The first thing to notice is that the proof of the paradox is infinitely long" (Forster, 1996). On the contrary, I will demonstrate that there is finite proof of the paradox in object language by applying universal quantification instead of a denumerable sequence of sentence operators. As far as the question of circularity is concerned, it is ambiguous: the answer cannot therefore be definite either, as James Hardy pointed out in his discerning paper (Hardy, 1995). If the question of circularity refers to the truth conditions of the list of formulae in first-order logic language, then the answer is definitely "No". If the question of circularity refers to the truth condition of the second-order logic formula that generates Yablo's list, then the answer is ambiguous, we will see later. Consider any large but finite initial segment of Yablo's sequence. This list can be perfectly simulated in spreadsheet computer software: the software will alert us if it is asked to execute a circular calculation. As the spreadsheet model clearly shows, the truth conditions of the finite list would be circular only if the list of

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[1] I want to thank Laurence Goldstein and Laureano Luna for important comments and suggestions. An earlier version of this paper, "What is demonstrated by Yablo's paradox?" was written jointly with the English philosopher Peter Fekete. Some points are better elaborated there, which I do not want to recapitulate in the present paper. See: http://www.andrasek.hu/ferenc/papers/yprx_en12rev.doc

natural numbers could be circular in opposition to the Peano axioms.[2] When the problem is simulated using spreadsheet software, it is visible on the screen that the finite list is not circular, because we do not get an alert message from the computer. Why would it be circular using an endless spreadsheet and Turing machine? Laurence Goldstein (2006) and Laureano Luna (2009) pointed out that the recursive definition of Yablo's sequence is in fact not circular but not well-founded (it has no base case). Graham Priest was the first to argue that Yablo's paradox is circular, and his argument proceeds in another direction and is connected to a fixed point construction. The third question also comes from Priest: "How can one be sure that there is such a sequence?" (Priest, 1997). I will prove that Yablo's sequence of statements does not exist if we restrict the domain of the sequence to natural numbers, although this fact should not mislead one into thinking that a sequence of signs does not exist. The source of the misunderstanding is that we inquire after the existence of statements and not formulae without interpretation. The meaning of elementary arithmetic terms is defined in the context of the paradox and is not freely interpretable without restriction. Discussing the existence of Yablo's sequence, J.C. Beall wrote: "Nobody, I should think, has seen a denumerable paradoxical sequence of sentences …" (Beall 2001). Fortunately, that power of imagination is not necessary: a little less is sufficient. Imagine "… At the gates of Heaven an infinite queue of people is tailed back" (Priest 1997), tirelessly putting on hats and taking them off again – we will understand why later.

Explication

To make matters clearer, I shall start the enquiry concerning the truth values of three second-order logic formulae.

The first second-order formula asserts the existence of a specific function, the range of which consists of well-formed formulae. They fulfil all the syntactical criteria for being sentences. The second formula claims the existence of an arithmetical function, and the third says there is an expedient sequence of true or false sentences. No doubt

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[2] The spreadsheet model is downloadable from: http://www.andrasek.hu/ferenc/models/pwsr-fin.xls

the first function exists with a range of well-formed formulae, but a paradox arises if we suppose that the existence of the first function supports the existence of a second and third function. The antinomy is that contradiction, and therefore falsity, follow from the existence of the second and third function, while, on the contrary, there is no problem with the first function, which, seemingly, proves the existence of the second and third functions.

In the original interpretation of Yablo's series, the domain of the sequence is isomorphic with the structure of natural numbers. I will ring the interpretation changes with the domain of Yablo's sequences that is set H. I will take H as the freely interpretable input of the formula that generates the list. Sometimes I suppose that H is isomorphic with natural numbers and H has a minimal element 0; sometimes I suppose it is infinite set of integers and it has one and only one a maximal element; and sometimes I suppose that H is finite. I will refer to (AR) when a line in the inference is a consequence of the A1, A2 or A3 axioms.

A1. Let '<' be a binary, asymmetric and transitive relation that defines a strict linear order in the domain of H.

A2. If $n \in H$ and $n \neq$ maximal element of H, then n has a unique successor that is n+1 and $(n+1) \in H$. This means there is no z in H such that $n<z$ and $z<n+1$.

A3. In some cases I suppose that there is a successor of n for every $n \in H$.[3]

Now consider the next three second-order logic formulae.

I. The first function:

(Ya) $\exists f \forall n: n \in H \rightarrow f(n) = \ulcorner S(n) \leftrightarrow \forall k\, (n < k \rightarrow {\sim}S(k)) \urcorner$

The function defined by (Ya) is f:

$f =_{df} \{<n, \ulcorner S(n) \leftrightarrow \forall k\, (n < k \rightarrow {\sim}S(k)) \urcorner > : n \in H\}$

Notice the citation function $\ulcorner \ldots \urcorner$ in the definiens. The initial segment of the function claimed by (Ya) is the next list:

$f(0) =$ '$S(0) \leftrightarrow \forall k\, (0 < k \rightarrow {\sim}S(k))$'

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[3] Jeffrey Ketland pointed out that paradox arises if '<' transitive and $\forall x(x \in H \rightarrow \exists y(y \in H \ \& \ x<y))$. (Ketland, 2004)

$f\,(1) =$ ‘$S(1) \leftrightarrow \forall k\,(1 < k \rightarrow {\sim}S(k))$’

$f\,(2) =$ ‘$S(2) \leftrightarrow \forall k\,(2 < k \rightarrow {\sim}S(k))$’

…

The domain of the function is the set of natural numbers – or a structure isomorphic with it – and its range is a denumerable set of well-formed formulae. Notice that if we apply Tarski’s T-schema for any member of the list, then we get an interpreted formula, a sentence that is true or false. Call attention that the definition of the function does not force us to apply any version of T-schema, because this list is a sequence of signs, a sequence of well-formed formulae between quotation marks. The definition of the function is not circular, and subsequently not mistaken. To prove this, for the sake of simplicity we can introduce a notation:

$G(n) =_{df} S(n) \leftrightarrow \forall k\,(n < k \rightarrow {\sim}S(k))$

Applying this, we get a simpler definition: ’$f =_{df} \{< n, \ulcorner G(n) \urcorner > : n \in H\}$’ that is not a circular definition of a function because no part of the definiendum occurs in the definiens.

II. In the second case, consider an arithmetical function so defined that it satisfies the following condition:

(Yb) $\exists f \forall n\colon n \in H \rightarrow (odd{=}f(n) \leftrightarrow \forall k\,(n < k \rightarrow even{=}f(k)))$

The sequence claimed by (Yb) is Y:

$Y =_{df} \{odd{=}f(n) \leftrightarrow \forall k\,(n < k \rightarrow even{=}f(n)) : n \in H\}$

Suppose H = the set of natural numbers. The initial segment of the presupposed function asserted by (Yb) is below:

(Y0) $odd{=}f(0) \leftrightarrow \forall k.\ 0{<}k {\rightarrow} even{=}f(k)$

(Y1) $odd{=}f(1) \leftrightarrow \forall k.\ 1{<}k {\rightarrow} even{=}f(k)$

(Y2) $odd{=}f(2) \leftrightarrow \forall k.\ 2{<}k {\rightarrow} even{=}f(k)$

…

(Yn) $odd{=}f(n) \leftrightarrow \forall k.\ n{<}k {\rightarrow} even{=}f(k)$

(Yn+1) $odd{=}f(n+1) \leftrightarrow \forall k.\ n+1{<}k \rightarrow even{=}f(k)$

The contradiction then arises following the argumentation:

*(1) $odd=f(n)$ (Yb) f suppose value of f is odd for n

*(2) $\forall k.\ n<k\rightarrow even=f(k)$ (Yn)

*(3) $n<n+1\rightarrow even=f(n+1)$ (2)

*(4) $n<n+1$ (AR)

*(5) $even=f(n+1)$ (3) (4)

*(6) $(\forall k.\ n<k\rightarrow even=f(k))\rightarrow(\forall k.\ n+1<k \rightarrow even=f(k))$ (AR)

*(7) $\forall k.\ n+1<k \rightarrow even=f(k)$ (2) (6)

*(8) $odd=f(n+1)$ (7) (Yn+1)

(9) $odd=f(n) \rightarrow.\ even=f(n+1)\ \&\ odd=f(n+1)$ *(1) (5) (8)

(10) $even=f(n)$ (9) (from elementary arithmetic)

(11) $\forall n.\ even=f(n)$ (10) by universal generalization over n in (10), since n was arbitrary.

(12) $\forall k.\ n<k\rightarrow even=f(k)$ (11) (AR)

(13) $odd=f(n)$ (Yn) (12)

(14) $even=f(n)\ \&\ odd=f(n)$ (10) (13)

Because there is no such number that is even and odd, therefore (14) is a contradiction, so the function f does not exist and $Y = \varnothing$. Yablo's antinomy is often called a ω paradox, because f function does exist for any large finite initial segment of Yablo's sequence, while in the case of an endless sequence, f does not exist. Matching the property of being odd to truth, and the even property to a false truth-value, we get the third second-order logic formula. Let $\sim S(n) =: even=f(n)$ and $S(n)=:odd=f(n)$.

III. Consider the affirmation of an endless sentence list:

(Yc) $\exists s\forall n\colon n\in \mathrm{H} \rightarrow (\, s(n) \leftrightarrow \forall k(n < k \rightarrow \sim s(k)))$

The sequence claimed by (Yc) is Y:

$Y =_{df} \{S(n) \leftrightarrow \forall k\ (n < k \rightarrow \sim S(k)) : n\in \mathrm{H}\}$

Let us apply the next notation:

$G(s, n) =_{\text{df}} \forall k\, (n < k \rightarrow \sim s(k))$

Putting into practice the above notation (Yc) turns into a simpler formula:

(YPG) $\exists s \forall n$: $n \in H \rightarrow (s(n) \leftrightarrow G(s, n))$

(2) $\forall n$: $n \in H \rightarrow (S(n) \leftrightarrow G(S, n))$ (YPG) S

Hence the set:

$YPG =_{\text{df}} \{S(n) \leftrightarrow G(S, n) : n \in H\}$

This schema, as a not recursive definition of S(n) list: $S(n) \leftrightarrow_{\text{df}} G(S, n)$, is circular, because the definiens – G(S, n) – comprises some part of the definiendum – S(n) – that is 'S' functor. In other case if '$S(n) \leftrightarrow_{\text{df}} G(S, n)$' is a recursive definition, then it must have a base case. This is what I referred to in the introduction. Based on the creative application of (YPG) schema, one can manufacture plenty of paradoxes (YPG = Yablo's paradox generator). Roy T. Cook recently presented two interesting applications of (YPG) schema:

$G_1(s, n) =_{\text{df}} \forall k\, ((n < k \rightarrow s(k)) \rightarrow \Phi)$

$G_2(s, n) =_{\text{df}} \forall k\, (n < k \rightarrow (s(k) \rightarrow \Phi))$

Cook proved that, similarly to the Curry paradox, every Φ sentence is derivable from (YPG) replacing G with G_1 or G_2. (Cook, 2009)[4] This is not very surprising, since in the framework of classical logic, everything follows from falsity, and (Yc) sentence is false. This can readily be proved:

Suppose H = the set of natural numbers. Then the initial segment of the sequence generated by (Yc) is below:

(Y0) $S(0) \leftrightarrow \forall k.\ 0<k \rightarrow \sim S(k)$

(Y1) $S(1) \leftrightarrow \forall k.\ 1<k \rightarrow \sim S(k)$

(Y2) $S(2) \leftrightarrow \forall k.\ 2<k \rightarrow \sim S(k)$

…

(Yn) $S(n) \leftrightarrow \forall k.\ n<k \rightarrow \sim S(k)$

(Yn+1) $S(n+1) \leftrightarrow \forall k.\ n+1<k \rightarrow \sim S(k)$

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[4] See my comments: http://www.andrasek.hu/ferenc/papers/cook7.doc

It was this series that Stephen Yablo invented. Notice that (Yc) is obviously false if we assign *s* variable for the predicate ‘to be equal to itself’ and if H is the set of natural numbers.

(1) $\forall n\colon n \in \omega \to (n=n \leftrightarrow \forall k\, (n < k \to {\sim}n=n))$

(2) $1 \in H \to (1=1 \leftrightarrow \forall k\, (1 < k \to {\sim}1=1))$ (1)

(3) $1 \in H \to (1=1 \leftrightarrow (1 < 2 \to {\sim}1=1))$ (2)

(4) $T \to (T \leftrightarrow (T \to F))$ (3) (from elementary arithmetic)

(5) F (4)

It is more complex to show that it is false to assign with any other predicate, such as ‘have a hat on’.[5] The contradiction arises as follows.

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[5] I will tell a story illustrating the problem. It is somewhat similar to Roy A. Sorensen’s and Graham Priest’s version of the paradox. In heaven, St. Peter is responsible for operations. Heaven is remarkably large but not limitless, and St. Peter has to ensure that it never becomes full. The only thing that St. Peter must do is to ensure that he never allows an infinite queue to get in. He is aware that the devil is always trying to find out how to set up an endless queue at the gates of heaven. On the other hand, there are sometimes quarrels at the gate. St. Peter introduces strict rules eliminating both problems:

a. everyone has one and only one line number in sequential order; and
b. everyone has a hat on if, and only if, nobody behind them has a hat on.

St. Peter has done his job well; he does not have to bother with the gate if there is an endless queue before the entrance. In the event that the queue is finite, St. Peter does have a job to do, which depends on the people in the queue complying with the rules. If there is only one soul before the entrance, that soul will argue thus: If there is someone with a hat on behind me, then I have no hat on; but nobody is standing behind me, so I have to have my hat on. What if there are two souls before the gate? The second one puts forward a similar argument, so has a hat on. The first one knows there is somebody behind her with a hat on, so she does not have a hat on. The situation with three souls is easy to determine. This can be illustrated as follows:

X
OX
OOX

Is the next figure a possible situation according to the rules?

OXX

No, because the second soul has a hat on only in the event that there is nobody with a hat on behind her. How can we formulate the general problem in the language of formal logic? Suppose every soul in heaven is represented by an ordinal number, the question is then: Is there a function f that satisfies the following condition?

For every n natural number (hatted=f(n) iff for every k (if n<k then not hatted=f(k)))

What if there is an endless queue before the gate? I will prove that there is no such f function in the domain of natural numbers. Suppose, on the contrary, that there exists such a function f_1 that satisfies the conditions. In this case, n has a hat on or not. Consider the first: hatted=f_1(n). If n has a hat on, then everybody behind her is bareheaded. Yes, but then soul n+1 close behind her also has a hat on. However, *(1) $S(n)$ - Suppose $S(n)$ is true (Yc) S

*(2) $\forall k.\ n<k \rightarrow \sim S(k)$ (Yn)

*(3) $n<n+1 \rightarrow \sim S(n+1)$ (2)

*(4) $n<n+1$ (AR)

*(5) $\sim S(n+1)$ (3) (4)

*(6) $(\forall k.\ n<k \rightarrow \sim S(k)) \rightarrow (\forall k.\ n+1<k \rightarrow \sim S(k))$ (AR)

*(7) $\forall k.\ n+1<k \rightarrow \sim S(k)$ (2) (6)

*(8) $S(n+1)$ (7) (Yn+1)

(9) $S(n) \rightarrow .\ \sim S(n+1)\ \&\ S(n+1)$ *(1) (5) (8)

(10) $\sim S(n)$ (9)

(11) $\forall n.\ \sim S(n)$ (10) because n was arbitrary

(12) $\forall k.\ n<k \rightarrow \sim S(k)$ (11) (AR)

(13) $S(n)$ (Yn) (12)

(14) $\sim S(n)\ \&\ S(n)$ (10) (13)

(15) If $\exists s \forall n\colon n \in H \rightarrow (s(n) \leftrightarrow \forall k\,(n < k \rightarrow \sim s(k)))$ then $\sim S(n)\ \&\ S(n)$ (Yc) (14)

(16) $\sim \exists s \forall n\colon n \in H \rightarrow (s(n) \leftrightarrow \forall k\,(n < k \rightarrow \sim s(k)))$ (15)

In other words, this means $Y=\varnothing$, so the sequence does not exist. It is worth noting that the contradictions disappear if:

a. H = negative integers

b. we extend the domain of (Yb), (Yc) functions to the first transfinite ordinal (ω) that is greater than any natural number. ($H = \omega \cup \{\omega\}$)

c. H = positive and negative integers $\cup \{\omega\}$

d. applying inverse relation of '<' relation.

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if she has a hat on, then the soul in front of her has no hat on, so n is bareheaded: not hatted=$f_1(n)$. We have a contradiction, thus our premise was wrong, therefore n soul must be bareheaded. However, this is true for every soul in the queue, thus nobody has a hat on. But anyone is permitted to have a hat on, which is again a contradiction. In this argument f_1 was arbitrary, so we have proved that no f function exists that fulfils the conditions.

This means that it is not the concept of infinity that is the root of the problem.[6] Notice that I did not apply any version of T-schema, omega rule, or fixed-point construction in this ontologically and formally parsimonious method of deductive inference. The derivation above therefore refutes J. Ketland's claim: "The derivation of an inconsistency requires a uniform fixed-point construction. Moreover, the truth-theoretic disquotational principle required is also uniform, rather than the local disquotational T-scheme. The theory with the local disquotation T-scheme applied to individual sentences from the Yablo list is also consistent" (J. Ketland, 2005). The antinomy demonstrates, as a proof by contradiction, only that such a series of formulae exists but not the assignment to true or false of those formulae as sentences. A map of formulae followed by consistent evaluation, known as an "evaluation map" (and not "model") does not exist in the framework of classical logic. On the other hand, if someone inquires as to the truth value of (Yc) second-order logic formula, its truth value is falsity, thus it has no model, as demonstrated by this antinomy. Let us now turn to a summary table of results from the analysis of (Ya), (Yb), (Yc) formulae. I will call an x term of a formula "input of a formula" iff x term (a predicate, function, sentence letter or individual constant) is freely interpretable in a certain context. In other words, a term with a rigid meaning is not an input of a formula.

The domain of the evaluation map (H set) is:	The evaluation map of:		
	(Ya)	(Yb)	(Yc)
Finite	Exists		
Denumerable infinite, and has no maximal element	Exists	Does not exist	Does not exist
Denumerable infinite, and has a maximal element	Exists		

Conclusion

It is a mistake to define a sequence using a non-recursive circular definition, or a recursive definition without base case, just as it is false to presuppose the existence of a

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[6] As far as I know, besides me only Eduardo Alejandro Barrio has understood this relationship. See: www.accionfilosofica.com/misc/1209050219inv.doc

non-existent series. There is a selection of set H – the domain of the series – when the existence condition holds, so the paradox is solvable. The solution is that set H has not only a minimal but also a maximum element that is the base case of the recursive definition. This is why Yablo’s paradox resembles Buridan’s paradox rather than the Liar paradox.[7] Let me explain in detail.

Philosophers understand paradoxes from two points of view. The first type of understanding seeks a cure for the disease; the second, as it were only taking note of the disease, wants merely to simulate the trouble and make an adequate model of the disease. The latter approach often leads to the amelioration of the patient, and is sometimes a source of effective therapy. Another question that I briefly address is whether the source of the paradox is a wrong definition, a mistaken language, or both of these. We come to a clearer understanding of the concept of paradox if we explain its usage in the context of related concepts. Let a set of formulae of a formal logic language

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[7] For those that do not know Buridan’s paradox:
"Twelfth sophism: GOD EXISTS AND SOME CONJUNCTION IS FALSE
The twelfth sophism is 'God exists and some conjunction is false'.
Let us posit that this is written on the wall and that there exists no other proposition than it and its parts. And then it is asked whether it is true or false.
We argue as before: for if it is true, then it follows that it is false; and if it is false, it seems to follow that it is true, for things are as it signifies, since its contradictory is false, namely this: 'God does not exist or no conjunction is false'.
Solution: we should say that it is false, and the argument is solved as before. For although things are as it signifies according to its formal signification, yet, things are not as would be signified by the consequent implied by it and the case, and, assuming it to be named by the proper name A, its contradictory would be this: 'No God exists or no conjunction is false or A is not true'.
Similar sophisms could be formed concerning disjunctive propositions, as 'A man is a donkey or some disjunctive is false', positing that there is no other disjunctive; and the same goes for exceptive [propositions], as for example, 'Every proposition other than an exceptive is true', positing that there are no propositions except this exceptive and two others, namely, that God exists and that a man is an animal; and thus also with exclusives, as when Socrates says: 'God exists' and Plato says: 'Only Socrates says something true', and nobody says anything else. Other sophisms can also be formed about the fact that it is possible for a proposition to be doubtful or not doubtful, known, or not known, believed or not believed."
John Buridan, Summulae de Dialectica (Summulae), an annotated translation with a philosophical introduction by Gyula Klima, New Haven: Yale University Press, 2001, Sophismata, c. 8, p. 980.

L be defined. We are talking about an evaluation map of a subset of L formulae if we define a certain function with the domain of formulae and range of truth values; we are talking about models of sets of formulae if there is an evaluation map that assigns every formulae of the set to true; and we are talking about a certain formal language that allows some terms to be formulated and forbids others. The following table – as an example – is self-explanatory: '⌜ ⌝' is a name-forming functor; S is the inverse of '⌜ ⌝' functor;'1,0' are truth values; 'T' is the 'true' predicate; Λ is the name of the Liar sentence; B is the name of a sentence of the medieval French philosopher Jean Buridan; G='God exists'.

Set of formulae	Context	Evaluation map	Model *
{p, ~q}	Classical logic	Defined for all four cases: $<0=\lvert p\rvert, 0=\lvert q\rvert> \rightarrow <0,1>$ $<0=\lvert p\rvert, 1=\lvert q\rvert> \rightarrow <0,0>$ $<1=\lvert p\rvert, 0=\lvert q\rvert> \rightarrow <1,1>$ $<1=\lvert p\rvert, 1=\lvert q\rvert> \rightarrow <1,0>$	There is: $1=\lvert p\rvert, 0=\lvert q\rvert$
{p, ~p}	Classical logic	Defined for both cases. $<0=\lvert p\rvert> \rightarrow <0,1>$ $<1=\lvert p\rvert> \rightarrow <1,0>$	Does not exist
Buridan paradox: $\{B=_{df}\ulcorner{\sim}(T(G) \vee T(B))\urcorner\}$ $\{S(B) \leftrightarrow_{df} {\sim}(T(G) \vee T(B))\}$	A mistaken closed language.	Evaluation map is a partial function because B is not an input of the formula. The definition is circular. $<1=\lvert T(G)\rvert> \rightarrow <0>$	Exists: $1=\lvert T(G)\rvert$ In case of $0=\lvert T(G)\rvert$ contradiction arises from the definition.
Strengthened Liar: $\{\Lambda =_{df}\ulcorner{\sim}T(\Lambda)\urcorner\}$ $\{S(\Lambda)\leftrightarrow_{df} {\sim}T(\Lambda)\}$	A mistaken closed language	Evaluation map does not exist because there is no input of the formula. The definition is circular.	Does not exist; contradiction arises from the definition.
Truth-teller paradox: $\{\Lambda=_{df}\ulcorner T(\Lambda)\urcorner\}$ $\{S(\Lambda)\leftrightarrow_{df} T(\Lambda)\}$	A mistaken closed language	Evaluation map does not exist because there is no input of the formula. The definition is circular.	Does not exist; tautology arises from the definition.
Yablo's paradox: $\{\exists s\forall n: n\in H\rightarrow(s(n)\leftrightarrow G(s,n))\}$	Second-order logic	<H is finite>→ <1> <H = natural numbers>→ <0> <H has max. element>→<1>	Exists, if H has a maximal element.
Yablo's paradox: $\{S(n)\leftrightarrow_{df}G(S,n) : n\in H\}$	First-order logic	Evaluation map is a partial; the definition is mistaken. <H is finite>→ <0 … 01> <H has max.elem.= H>→<0…01>	Exists, if H has only one element.

* Applying finite formal logic language fragments every formula and its evaluation map can be simulated in spreadsheet software (e.g. Excel).[8]

Let us explain the moral of the table above. The paradox is not a lonely warrior but a soldier in an army. In certain language contexts an H set of formulae is not paradoxical if in any of non-empty domain of discourses; any interpretation of inputs (predicates, functions and names); and any evaluation map of sentence letters, consistently assigns truth values to all the sentences in H. Just as, vice versa, if:

1. not every evaluation map of sentence letters; or
2. not every interpretation of predicates, functions and individual names; or
3. not every non-empty domain of discourse assigns consistently truth values to all the sentences, then we are talking about logical paradox.

(In the case of Yablo's paradox, this means that if only one sentence in the sentence series is ambiguous in a certain selection of the domain of discourse, then the whole series is paradoxical.) A paradox is partially solvable if the evaluation map partially ensures the truth values of the formulae (e.g. Buridan's paradox). It is unsolvable if there is no consistent evaluation map at all (e.g. the Liar paradox). The paradox is eliminable if we know how to block the composition or derivation of the paradox applying a specific formal logic framework. Logical paradox cannot be formulated in the framework of classical logic, thus we have to apply a mistaken semantically closed formal logic. Let classical logic be characterised by the usual formation and inference rules that we extend in three axioms below. In this way I outline a mistaken closed formal logic language that has enough assertibility power to formulate well-known paradoxes.

(AC1) $\forall \ulcorner\urcorner \forall x\ \forall s$ (If s is a sentence of L_1 language, and there is x sentence name of L_1 that $x=\ulcorner s\urcorner$, then x is a term of L_1);
(AC2) $\forall \ulcorner\urcorner \forall x$ (if $\exists s\ x=\ulcorner s\urcorner$ – that s is a sentence of L_1 language – then 'T(x)' is also a sentence of L_1);

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[8] See my spreadsheet models: http://www.andrasek.hu/ferenc/papers.htm

(AC3) $\forall \ulcorner\urcorner \forall x(T(x) \leftrightarrow \exists s\, (x=\ulcorner s\urcorner \;\&\; s\in L_1 \;\&\; s))$ – a feasible formulation of Tarski’s T-schema.

Both the Liar and Buridan paradoxes are plainly derivable from the axioms of this closed formal language. In the case of sentence B we have to select $1=|T(G)|$ evaluation map, otherwise we become embroiled in a contradiction. However, the Liar paradox does not allow us a loophole. Nor does Yablo’s sequence if we restrict the domain of discourse to natural numbers. Semantically closed formal language is not needed to formulate Yablo’s series, because it can be formulated in second-order logic. Ringing the changes of the domain of Yablo’s series, the result is similar to Buridan’s paradox, because both of them have a partial evaluation map. Burdian’s paradox is a “proof” of the existence of God – a maximal element of our world; Yablo’s paradox is a proof of the existence of the maximal element of H.

This is the penultimate sentence of my paper, which is true if and only if none of my foregoing sentences is false.

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