Why is quantified modal logic dubious?

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2011

Quine devised many arguments against quantified modal logic; however, the aim of this study is to revitalize the topic. The new argument is based on FOL (first-order logic) elaborated with functions and axioms of identity.

Let us consider the following statements:

(1) The temperature of the water in the pot on the cooker is 100 °C.
(2) The temperature of the water in the pot on the table is 18 °C.
(3) The ambient temperature of the pots is 18 °C.
(4) The temperature of the water in the pot on the table after a certain time is equal to the ambient temperature.

Let us translate these four sentences to FOL. Following the language habits of engineers and physicists, instead of predicates, we will use functions and abstract entities. These abstract entities are nothing but the possible outcomes of experiments and measurements. Let us define three individual constants in this way:

c:=the water in the pot on the cooker
d:=the water in the pot on the table
e:=the ambient temperature of the pots

We can then obtain the following formulas, where "f" is a function that defines the temperature of physical objects in its argument. Let us denote the measurable properties–mass, temperature, distance, voltage, etc.– as "characteristic". Every characteristic has more then one possible value, and is a function of space-time. At this point, for the sake of simplicity, let us disregard time.

(1) 100 °C = f(c)

(2) 18 °C = f(d)
(3) 18 °C = f(e)
(4) f(d) = f(e)

We know from secondary-school physics lessons that (1) and (4) are necessarily true, but that (2) and (3) are not. This is so because the necessary truth of the first and fourth sentences does not depend on the logical structure of those sentences, but on whether or not those sentences follow from physical laws. Usually, "f(c)" is called a rigid designator, because in every possible world and in every history of that world, under standard conditions for ambient temperature and pressure, the temperature of boiled water is 100 °C. However, it is presumed that the rigid or non-rigid designator character of names–in our case, "c,d nad e"–does not provide a plausible explication of the necessity of physical laws. The physical law is the root of the rigid character of that name, and not vice versa. Neither f(d) nor f(e) are rigid designators, because the laws of physics do not preclude that the ambient temperature is other than 18 °C. In contrast to non-rigid character of "f(d)" and "f(e)" names, sentence (4), which is composed of "f(d)" and "f(e)", is inferred from a physical law, and thus, it is not only contingently true, but necessarily true. On the other hand, it must be noted that the rigid name character of the individual "c" does not follow from its grammatical role, but from its meaning. However, neither the meaning of a name nor the rigid or non-rigid character of names has any influence in formal logic inferences. In our case, the necessary truth of (1) and (4) does not follow from the logical structure of those sentences, but from extra-logical facts, namely physical laws. On the contrary, in quantified modal logic (QML), if an identity sentence is true, then that sentence is necessarily true:

(1)	$\forall x \forall y. x = y \rightarrow Lx = y$	
	(where L is the necessity operator.)	
(2)	$a = f(b) \rightarrow La = f(b)$	(1)
	(where f is a function.)	

The step from (1) to (2) is an admissible derivation step in FOL. If it is not an admissible step in QML, then QML is not a formal logic, because the meaning of the terms has significance in the inference.

(A) Let us assume the following axiom: for every F one-place predicate, there is an f function such that $\exists y \forall x. F(x) \leftrightarrow y = f(x)$.
For example: $Red(x) \leftrightarrow red = the\_colour\_of(x)$

(3)	$F(b)$	(premise)
(4)	$F(b) \leftrightarrow a = f(b)$	(A)
(5)	$a = f(b)$	(3)(4)
(6)	$La = f(b)$	(2)(5)
(7)	$LF(b)$	(4)(6)
(8)	$F(b) \rightarrow LF(b)$	(3)(7)

This means that every proposition that has a logical form $F(b)$ is necessarily true or false, "but this (16) postulate annihilates modal distinctions; for we can deduce from it that 'Necessarily p' holds not matter what true sentence we put for 'p'.". Quine (1960: *Word and object*, MIT, pp.196,197) Assembly logic with various axioms of physics that are translated into logical language, a necessarily true formula follows form this enlarged set of axioms. However, the "inference" relation is a relation that ranges over a domain of formula names and not a truth function between formulas: it mentions the formulas and does not use them. This is why QML is dubious.
According to M. J. Cresswell: "...in a modal language most expressions which look like function expression actually turn out not to be. ...For this reason it may well be preferable to avoid altogether the use of function symbols in modal predicate logic." G. E. Hughes, M. J. Cresswell (1996: *A new introduction to modal logic*, Routledge, p.328) For engineers and physicists, a formal language including functions is indispensable, so for them it is preferable to avoid modal predicate logic. This means that modal predicate logic is dispensable for natural sciences. Why then is it so important for naturalist philosophers?