What is Truth?

Ferenc András

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"What is truth?" Pilate asked the Master, but he didn't even wait for an answer, because he didn't believe in the truth. It was wrong of him.¹

You should know that the Father collects human sentences in three bags. In the right-hand bag are true, declarative sentences; in the left-hand bag are false sentences; while unclear or misleading sentences are in the middle bag. All the sentences are written on pieces of paper, and each piece of paper is marked with a name, carefully, in strict order, so that no sentence is left without a name, there are no two sentences with the same name, and each sentence occurs only once. The Father does not allow the angels to sit around with their feet dangling from the clouds. Rather than being idle, their task is to take the confusing sentences out of the middle bag and to attempt to add explanatory notes that give them a clear, solid meaning. If they succeed, the corrected sentence is repeated before the Father, who places it in the right-or left-hand bag.

Once upon a time, Lucifer wanted to annoy the Father, so he smuggled the following sentence before the angels:

 (λ) This sentence, named λ , which you are now reading, will confuse you: you won't be able to grasp it, because it's not true, or its truth is ambiguous, or perhaps it doesn't even have a truth value at all.

The angels read the note and understood what it meant, because it was perfectly clear what this sentence was about. The devil's sentence was about

¹A shorter version of this text appeared earlier in a Hungarian-language philosophical blog: https://namitgondolsz.blog.hu/2015/02/21/isten_es_az_ordogi_mondat

itself. It didn't state the number of letters it contained, or the language it was written in, but said something about its truth. It said something strange and unusual, which was apparently simple and comprehensible—and perhaps it really was simple. The only question was, which bag should it go in? The angels sometimes made their suggestions discreetly, on the back of the piece of paper, but in this case they began arguing with each other, quarrelling so loudly that they woke the Father, who was napping on his throne. He scolded them: "Don't quarrel, just bring me the sentence and I'll decide. If what the sentence says is really clear and understandable, it must be either true or false, there's no third possibility." Or so he thought. The truth of a sentence depends on the facts, and the fact is that the truth of the sentence is not clear, so it is as it says. But if the reality is as the sentence says, then the sentence is true, thus sentence λ is true. So we'll put it in the right-hand bag. Or will we? If the sentence λ is true, then it is as it says. But what it says is that it is not true, or that its truth is not certain, while the right-hand bag contains only incontestable truths such as "Snow is white." This sentence isn't like that, so it can't be put in the right-hand bag. It will have to go in the left-hand bag, together with the false sentences, because there is some sense in the sentence, it's just a little tricky. Thus it isn't true that the sentence you're reading is confusing, that you can't grasp it because it isn't true or its truth is ambiguous, or it may not have a truth value. The Father had almost started looking at the next sentence when he heard Lucifer chuckle. He thought again. If the sentence is false, then it has a clear truth value that is certainly not true. However, if it is not true, then how is it possible that what the sentence says corresponds to the facts: not true, or its truth is ambiguous ... Its truth value is not uncertain, but the other alternative is fulfilled—that is, that the sentence is not true. In which case, the entire sentence must be true and not false. We have introduced a contradiction, but now everything is clear, the sentence is neither true nor false, but is a complete absence of reason, chaos, and that is a certainty. But no! This is what the sentence says, so it is telling the truth and is therefore true. However, this has already been explored once, we're back where we came from, we've got ourselves caught in a vicious circle. But then the Father looked up from his throne and smiled quietly.

What had happened was that a certain soul by the name of Alfred Tarski had arrived for the celestial brunch and was speaking quietly to Cantor and Russell, explaining that it was not right for the Father to have only three bags. An infinite number of floors should be built above his throne, with an endless number of bags. Georg, you bring the raw materials for the infinite number of floors; and Bertrand, you build the infinite levels. And it was so. When the Father realised that the bags on the first floor contained sentences about the sentences on the ground floor, and the bags on the second floor contained sentences about those on the first floor, he gently tossed Lucifer's sentence up one floor into a bag. When he saw this, Lucifer turned away miserably. But why did the evil one concede defeat? Was Lucifer's sentence true or not? Can a convincing answer be found regarding the nature of truth? Let me explain briefly.

Language L_1 is an object language relative to language L_2 . Language L_2 is a meta-language in relation to language L_1 . The connection between an object language and a meta-language is a relation. For each object language there is a higher-level meta-language. The lowest-level object language, which is only about properties of the physical world, contains no semantic predicates. This language is denoted by L_0 .

Lucifer's sentence, called λ , is a part of the language L_1 . The Father evaluates this sentence using the language of the seraphs, a higher-level metalanguage called L_2 . In these higher-level languages we apply the True₂ and False₂ metalanguage semantic predicates, the extensions of which make up sentences with the name of the object language (L_1) , thus Lucifer's sentence λ is a member of this language. The index sign of the metalanguage predicates refers to the language level. At L_1 metalanguage level, λ is neither True₁ nor False₁. Thus in the language L_2 —applying the language of set theory—the Father claims that $\lambda \notin$ True₁ and $\lambda \notin$ False₁. The name of the last sentence in the higher-level language L_2 is β ($\beta = `\lambda \notin$ True₁ and $\lambda \notin$ False₁'). According to the definition of True₂ in L_2 , the Father's sentence β is True₂, which is convincing because the Father always tells the truth. This can be summarised in a Table 1.

Following Tarski, according to the convention (T): if p is a sentence at n language level, and the name of the sentence p is x (at the n + 1 language level), and p' is the translation of p at level L_{n+1} , then (at the n + 1 language level) x is True_{n+1} if and only if p'. In other words, $x - \text{True}_{n+1}$ at the n + 1 language level, when sentence x refers precisely to the facts. It is essential for the theorem to be valid vice versa, thus if we make the claim regarding p, and x is the name of p, then x is true. The definition of truth in formal languages

Language	Sentence	Truth predicate
L_0	The snow is white.	
L_1	$\lambda = \lambda$ sentence will confuse you, you won't	$\lambda \notin L_0$ so it has
	be able to grasp it, because it's not true, or	no $Truth_1$ value.
	its truth is ambiguous, or maybe it doesn't	
	even have a truth value at all.'	
	$\alpha =$ 'The snow is white.'	$\alpha \in L_0$ so it has
		$Truth_1$ value,
		and α -is-Truth ₁
L_2	β ='Lucifer's sentence is neither True ₁ nor	β -is-True ₂
	$False_1.'$	
L_3	$\gamma = $ 'It is True ₂ that Lucifer's sentence is nei-	γ -is-True ₃
	ther $True_1$ sem nem False ₁ .'	
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Table 1: Language levels

must be formally correct and, at the same time, materially adequate; in other words, it must imply all equivalences of the form (T). Imre Ruzsa noted that Tarski's concept can also be formulated in such a way that the truth bearers are propositions, not sentences. The truth theories of Kripke, Barwise, Belnap and others do not contradict Tarski, but are alternatives to Tarski's theory. In Tarski's theory, the relationship between language and reality can be described at the meta-language level.

Exercises

Consider the following sentences. Which is true, which is false, and which is paradoxical? The term "paradox" in this case refers to unstable, alternating, or not true and not false logical sentences. In this case, a sentence is not well formed if its truth value depends on its own truth value.

- (1) This sentence is nonsense, it has no meaning.
- (2) This sentence is paradoxical.
- (3) This is a not well-formed sentence.
- (4) This sentence is not true or it is an English sentence.

- (5) This sentence is not true or it is a French sentence.
- (6) This sentence is not true or it is not a well-formed sentence.
- (7) This sentence is not true or it is a well-formed sentence.

The sentence "The present king of France is bald" has no truth value (following Frege). It is questionable whether the sentence property 'paradoxical' differs from a simple value gap. Another open question is whether it is possible to have a logically partially erroneous or confusing sentence that is true (e.g.: "vgii f fifif i or snow is white"). The question is confusing, because if the sentence begins with a meaningless part ("vgii f fifif I"), then how do we know that the rest ("or snow is white") is meaningful and not without sense in some other way? The meaningless part perhaps brings the sense and meaning of the sentence as a whole into question. If (6) is True₁ then (6) is not paradoxical, but some True₁ sentences are not well formed. Otherwise, sentence (6) is a paradox (not True₁ and not False₁).

Which of the seven statements above can be considered as the sentence of the strengthened liar paradox?

The solutions: https://sht.andrasek.hu/what-is-truth-excercises.xlsx