A note on the Ship of Theseus puzzle

draft

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Introduction

While driving home on a long, winding road, Theseus notices a car repair shop by the side of the road. As his car brakes have been in poor condition for some time, he decides to get them repaired.

> "What's the problem with the car, Sir?" asks the mechanic.
>
> "The brakes are squealing, and the stopping distance is getting dangerously long."
>
> "Were the brakes working well when you bought the car?"
>
> "Yes, they worked perfectly, but since then the brake pads have worn very thin."
>
> "You must be mistaken, Sir. When you purchased the car, the brakes worked well because the brake pads were in good condition. The day after you bought the car, the brake pads wore to an invisibly small extent, such that the condition of the brakes was the same as on the previous day, thus the brakes were good on the second day. On the third day, the brake pads again wore slightly compared to the previous day, but the condition of the brakes was the same as on the preceding day, meaning the brakes were good. This has been the case each day until today. The brake pads are therefore as good today as they were yesterday. There is nothing wrong with the car brakes. They're still in good condition."

Where is the flaw in the mechanic's philosophical reasoning?

1. The wearing down of the brake pads is a cumulative process. If the rate of wear on the first day is Δ, then on the second day it is $2 \times \Delta$, and on the n-th day it is $n \times \Delta$.

2. We must compare the present condition of the brake pads to their original condition – that is, to the standard condition – not to the condition on the preceding day. If the rate of wear reaches a particular limit, then the condition of the brake pads cannot be described as "good". When a certain unacceptable level of wear has been reached, the brake pad is no longer good. A good mechanic is aware of this limit, even without the aid of a philosophical treatise.

3. If we compare the level of wear of the brake pad to the condition on the preceding day, we get a similarity (also called a proximity) relation, which, in mathematical language, is a tolerance relation. (The tolerance relation was invented by E.C. Zeeman in 1961 in modelling visual perception. Ross 2004) If we compare the wear on the brake pad with the original state, then we get an uniformity relation, which, in mathematical language, is an equivalence relation.

The problem presented here is similar to the Ship of Theseus problem, although in the latter the question concerns the replacement of planks or other worn parts of a ship, rather than worn brake pads. If the amount of decay in the first year is Δ, it will be $2 \times \Delta$ in the second year, and $n \times \Delta$ in the n-th year. The change, when compared to the previous state, defines a tolerance relationship that is a reflexive and symmetrical, but not a transitive, relationship, and consequently contradicts identity. Identity is, in fact, a transitive relationship. However, if we compare the change to the standard, we get an equivalence relationship, which is consistent with the assumed self-identity. As long as we have replaced fewer than half the ship's planks, the ship temporal slice is uniform with the original ship temporal slice, but beyond that, it is not. If we replace more than half the ship's planks, then the Ship of Theseus ceases to exist, although its planks may survive as mementos or parts thereof. As long as the number of planks changed remains below the limit, the ship exists and is identical to itself. Depending on the framework, the ship is therefore an entity that exists over time or it is a four-dimensional object that consists of temporal slices.

The exhibited ship

There are different criteria for the identity and persistence of the ship during the voyage, and in the case of the exhibited ship. Even if all parts of the ship are replaced during the voyage, it does not imply any problem of identification. Theseus is sailing home on the ship, and that is the identity criterion for the ship. If he were to switch ships during the voyage, there would be

no point talking about the ship. During the voyage, his ship is similar to a living body, because every replaced part becomes part of Theseus's ship, and the parts that are detached and thrown away are no longer parts of the ship.
The situation is different in the case of an exhibited ship, where the aim is a constant condition and an unchanged existence. The Ship of Theseus is a means of transportation during the voyage, while the exhibited ship is no longer a vessel, but a memorial. The exhibited ship was Theseus's ship at an earlier time, but is now a memorial. Since his return, Theseus has acquired another ship. On the other hand, we have to decide the extent of decay after which the exhibited ship can no longer be considered the descendant of the famous ship. Once this limit is passed, we may claim similarity to the original ship, but would have to deny uniformity with the original ship. When defining the similarity and uniformity of the exhibited ship's temporal slices, we must suppose measurable criteria for gradual decay between the current and the previous condition, and the current and original condition of the ship. (In the paragraphs below, by 'Ship of Theseus' I mean the exhibited ship.)

Following Amie L. Thomasson's considerations (Thomasson 2007), we have the following alternatives: The application condition – Is it a vessel or a memorial? The identification criterion – Is it Theseus's ship or was it Theseus's ship? The co-application condition – Is it the same ship as before, or not?

Suppose the parts of the exhibited ship have decayed and gradually been replaced in 10 steps. At temporal slice '10', a maximum of 10% of the original parts have been replaced; and at temporal slice '20', a maximum of 20% have been replaced, etc. Based on the above, the set of all possible temporal slices of the ship that has gradually decayed can be denoted as: $S = \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$, $S_T =$the set of uniform temporal slices. The set of discrete moments in the history of the Ship of Theseus is T. I do not suppose the continuous existence of the exhibited ship. It may happen that it is disassembled and later reassembled as a memorial. The following definition can then be used:

Def. 1. an $x \in S$ ship temporal slice resembles $y \in S$ temporal slice if and only if a maximum $d\%$ divergence is present in its parts The value of d should be determined in the light of practical considerations and non-a priori principles. In this case, I have selected $d = 30$. In Table 1, the sign '1' denotes similar ship temporal slices and '0' denotes non-similar temporal slices.

$x \approx y :=$ x and y are similar temporal slices

Table 1: similarity

$\approx$	10	20	30	40	50	60	70	80	90	100
10	1	1	1	0	0	0	0	0	0	0
20	1	1	1	1	0	0	0	0	0	0
30	1	1	1	1	1	0	0	0	0	0
40	0	1	1	1	1	1	0	0	0	0
50	0	0	1	1	1	1	1	0	0	0
60	0	0	0	1	1	1	1	1	0	0
70	0	0	0	0	1	1	1	1	1	0
80	0	0	0	0	0	1	1	1	1	1
90	0	0	0	0	0	0	1	1	1	1
100	0	0	0	0	0	0	0	1	1	1

Note that the tolerance relation is reflexive and symmetric, but not transitive, unlike the equivalence relation, which represents uniformity. Tolerance is weaker than equivalence; it does not need to be transitive. In our case, two ship temporal slices are similar if the difference between them is up to 30%. Following the definition of the tolerance relation, $a \approx b$ and $b \approx c$ does not entail that $a \approx c$, in contrast with the identity or uniformity relationship.

We represent the uniformity of ship temporal slices by applying the equivalence relation. It is advisable to select criteria of uniformity such that, even when the ship is disassembled, it is not possible to create rival renovated ships. If we allow that $d = 50\%$, there might be two of the same ship. Both ships would contain a share of half the original parts, thus we would not be able to decide which to consider the original ship's successor. I then introduce the next definition:

Def. 2. x ship temporal slice is uniform to y temporal slice based on Ship of Theseus:= x and y temporal slices' divergence from the original ship is less than 50% or $x = y$.

$x \cong y :=$ x and y are uniform temporal slices

Table 2 represents the exhibited ship. The Ship of Theseus survives some

Table 2: uniformity

≅	10	20	30	40	50	60	70	80	90	100
10	1	1	1	1	0	0	0	0	0	0
20	1	1	1	1	0	0	0	0	0	0
30	1	1	1	1	0	0	0	0	0	0
40	1	1	1	1	0	0	0	0	0	0
50	0	0	0	0	1	0	0	0	0	0
60	0	0	0	0	0	1	0	0	0	0
70	0	0	0	0	0	0	1	0	0	0
80	0	0	0	0	0	0	0	1	0	0
90	0	0	0	0	0	0	0	0	1	0
100	0	0	0	0	0	0	0	0	0	1

gradual change. The equivalence classes in the table contain the uniform temporal slices. The '10,20,30,40' temporal slices are uniform with the original condition of the exhibited Ship of Theseus, and any two elements of the equivalence class are the same ship, just at different moments. The other elements are not the same ship: they are different because their divergence from the standard is more than 50%. We treat '50,60,70,80,90,100' temporal slices as the ruins of the ship. Based on this, the Ship of Theseus in the framework of perdurantism can be defined in this manner:

$$\text{Ship of Theseus}_{perdurantism} := \langle T, S_T, R \rangle, \text{ where } R \subseteq T \times S_T$$

$$\forall t \forall x . t \Re x := t \in T \wedge x \cong 10$$

(x is Theseus's ship temporal slice at $t \in T$ time if and only if x is uniform to the original exhibited ship. T set map to S_T equivalence class.)

We can formulate certain ideas more easily in the framework of endurantism than in the framework of perdurantism. They both have advantages and disadvantages. However, using the language of classical logic, we cannot formulate the claim about the exhibited ship ceasing to exist – the ship is destroyed at a given time and no longer exists after a time – in the framework of endurantism. The individuum name 't_h' denotes the exhibited ship in the framework of endurantism.

$$y = t_h := \forall z \forall t \in T(z = \text{the condition of } y \text{ at } t \rightarrow z \cong 10)$$

(y is Theseus's ship if and only if for every $t \in T$ time and z ship condition, if z is the condition of y at t, then z is uniform to the original exhibited ship.)

If $\sim \exists x \exists y (x \cong y)$ then there is no ship, there are no artefacts (Mereological nihilism: Peter van Inwagen).

If $\forall x \forall y (x \cong y \rightarrow x = y)$ then ordinary objects never survive any changes (Mereological essentialism: Roderick Chisholm, Joseph Butler).

If $\forall x \in S\ \forall y \in S (x \cong y)$ then the original ship is identical with the renovated ship (Edward Jonathan Lowe).

Applications

Until any new event happens to the object, in the framework of perdurantism we do not know what that physical object is. We can resolve this problem in the case of the exhibited ship by assuming that there exists a $\Re$ function relation, although we can only partially know the $\Re$ function relation – that is, we cannot predict the history of the ship. In this way, we can state a few facts about the ship.

(1) If there is a Ship of Theseus at all at one time, then there is only one.

(perd 1) $\forall x \forall y \forall t ((t \Re x \wedge t \Re y) \rightarrow x = y)$

(end 1) $\forall x \forall y ((x = t_h \wedge y = t_h) \rightarrow x = y)$

Applying Leibniz's Law the identity of indiscernibles in framework of second order logic:

(end 1*) $\exists t \in T \forall \Psi (\Psi x t \rightarrow \Psi t_h t) \rightarrow x = t_h$
(The converse of the principle–the Indiscernibility of Identicals–is an axiom schema of classical First-order Logic.)

(2) The exhibited ship was the Ship of Theseus.

(perd 2) $\forall t \forall x (t \Re x \rightarrow \text{was-the-ship-of-Theseus}(x))$

(end 2) was-the-ship-of-Theseus(t_h)

(3) The condition of the exhibited ship becomes worse and worse.

(perd 3) $\forall t_1 \forall t_2 \forall x \forall y((t_1 \Re x \wedge t_2 \Re y \wedge t_2$ is the next moment to $t_1)$
$\rightarrow y$-is worse condition than-$x)$

(end 3) $\forall t_1 \forall t_2 \forall x \forall y((t_2$ later then $t_1 \wedge x =$ the condition of t_h at $t_1 \wedge$
$y =$ the condition of t_h at $t_2) \rightarrow y$-is worse condition than-$x)$

(4) After a certain time, the exhibited ship ceases to exist.

(perd 4) $\exists t_1 \forall t_2 (t_2$ later then $t_1 \rightarrow \neg \exists x (t_2 \Re x))$

Summary

The key to the puzzle lies in distinguishing the tolerance, equivalence and identity relations. The first two relations are mathematical relations, and it is up to you how to interpret them. The identity relation is a logical constant of classical logic, and it is forbidden to interpret a logical constant. On the other hand, the decay of an ordinary object is a cumulative process. There must therefore be a limit in the decay process, and when this limit is crossed, the famous ship ceases to exist. The self-identity of ordinary objects over time can be expressed by the uniformity of the temporal slices of objects. The definition of the uniformity relation is based on variance from the standard, not variance from the previous state of the object. Uniformity is based on the point of view that temporal slices of the object are slices of the same object, and that the rate of change will remain within the given limit. Ship temporal slices may differ in certain characteristics at different times. There are no a priori principles of self-identify that are independent from ordinary or scientific practice. Language usage is a community decision. Theodore Scaltsas wrote:

> I have argued that there are at least three major criteria that may serve as sufficiency conditions(or at least as conditions) for determining whether the identity of an object or an artefact is recovered in various instances of repair, reassembly, reconstruction, etc., of an object or artefact that has ceased to be. I claim there that the sufficiency condition might be the spatio-temporal continuity of the form of the object, or the identity of the parts, or the identity of the matter of the object, and further, that the origin and the history of the object might play an integral part in determining whether the initial object is recovered or not. What I wish to emphasize further and argue for here is that there is no sharply defined hierarchy of sufficiency conditions, so that in cases of conflict we are not always in a position to determine

> whether the new object is identical to the initial one or not. The reason is that the cases of conflict are so rare in everyday life that we have not had the need to compare the various sufficiency conditions and determine the relative strength of each in cases of conflict. . . . What I have tried to show here is that we can create many paradoxes of the Theseus' ship type, precisely because we do not employ a sharply defined code of sufficiency conditions and overruling hierarchies between these conditions when deciding questions of artifact reidentification after some ceasing-to-be of an artifact and recovery. Discovering the lack of such a reidentification code is discovering something very significant about our intuitions and habits concerning decision making on artifact identity. But this does not mean that in the face of the lack of such a code we can assume or impose a code which we happen to find appealing and convincing; only the need for making such decisions in everyday life will force us to develop a functionally acceptable code of artifact reidentification. (Scaltsas 1980)

The relationship describing the uniformity of the temporal slices of the ship is a formal logical equivalence relation, exactly the same as the relationship of identity. As long as the object exists, the physical object's time range is mapped to the subset of an equivalence class of equivalence relation based on uniformity. When we talk about the self-identity of a physical object over time, we are talking about a series of temporal slices in the time range of the object's existence, which are elements of an equivalence class.

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