A few comments on Roy T. Cook’s “Curry, Yablo and duality” paper

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In his interesting paper, Roy T. Cook (2009: *Curry, Yablo and duality*, Analysis Vol 69, Number 4, October) investigates two infinite sequences of formulas which are similar to Yablo’s series. He claims something about the formulae and gives us proof of them. My reply consists of two parts. The first part (1–3) is rather polemical, but in the second part (4), I will confirm in another way, what Cook claims is true.

1. Cook formulates Tarski’s T schema this way:

“T(<Λ>) ↔ Λ” p. 612

He noticed: “Here ‘<…>’ is any appropriate name-forming device, such as Gödel coding. … where T is a truth…” It is clear that Λ above is a sentence variable or a sentence schema and not a sentence constant like the Liar sentence.

2. How to formulate the Liar sentence? This is Cook’s answer:

Λ: ~T(<Λ>)

What does the colon mean in the above? In some contexts, a colon introduces a list or a set, referring to an individual term or an object, or defines a predicate, sometimes introducing the logical consequence, and there are many other usages. In our case, Λ is a sentence – not a name – and ‘~T(<Λ>)’ is also a sentence, so this colon means ‘if and only if’, or the introduction of a definition. Later, however, Cook introduces a definition using the ‘$=_{df}$’ sign: “If we define negation in terms of conditional a primitive absurdity constant ‘⊥’: $\sim\psi =_{df} \psi \rightarrow \bot$” (p. 613) we have no other choice than the first option. The letter ‘Λ’ is a variable or schema name in Cook’s formulation of T schema, but in the formulation of the Liar paradox it should be a constant, not a variable. It would be better to use different symbols for different grammatical roles.

3. There is a typo on p. 618:

(f) the universal quantifier was missing: $(\underline{\forall} m > j+1)\ (T(<Sm>) \rightarrow \Phi)$

Later he says: "Applying modus ponens to (e) and (g) gives us the desired result: Φ"
Sorry, I can not see the modus ponens: $\{T(<Sj_{+1}>) \rightarrow \Phi,\ T(<Sj_{+2}>)\} \Rightarrow \Phi$??

4. Now I demonstrate an alternate proof of Cook's main theorems by applying an arithmetic translation. Let n and m be natural number variables, S a unary predicate, and α a unary predicate variable.
Cook's first claim (Dual Yablurry), without semantic predicates and replacing colons with '↔' symbols is:

(1) $S(1) \leftrightarrow (\forall m\ (m>1 \rightarrow S(m)) \rightarrow \Phi)$

(2) $S(2) \leftrightarrow (\forall m\ (m>2 \rightarrow S(m)) \rightarrow \Phi)$

....

(n) $S(n) \leftrightarrow (\forall m\ (m>n \rightarrow S(m)) \rightarrow \Phi)$

….

In fact the sentence series above follows from:
$\exists\alpha\forall n\ (\alpha(n) \leftrightarrow (\forall m\ (m>n \rightarrow \alpha(m)) \rightarrow \Phi))$ second order logic formula.
The next inference has only five lines.

$*$(1) $\exists\alpha\forall n\ (\alpha(n) \leftrightarrow (\forall m\ (m>n \rightarrow \alpha(m)) \rightarrow \Phi))$

$*$(2) $\forall n(2*n+1 \text{ is odd} \leftrightarrow (\forall m\ (m>n \rightarrow 2*m+1 \text{ is odd}) \rightarrow \Phi))$ (1)

I substituted α with the '2*①+1 is odd' predicate, but it also works with the '①=①' predicate.

$*$(3) $(\text{True} \leftrightarrow (\text{True} \rightarrow \Phi))$ (2) from elementary arithmetic

$*$(4) Φ (3)

(5) If $\exists\alpha\forall n\ (\alpha(n) \leftrightarrow (\forall m\ (m>n \rightarrow \alpha(m)) \rightarrow \Phi))$ then Φ (4)

Q.E.D.

Second claim (Yablurry):

$\exists\alpha\forall n\, (\alpha(n) \leftrightarrow (\forall m\, (m>n \rightarrow(\alpha(m) \rightarrow \Phi))))$

It is easy to prove the second formula in a similar way.

* (1) $\exists\alpha\forall n\, (\alpha(n) \leftrightarrow (\forall m\, (m>n \rightarrow(\alpha(m) \rightarrow \Phi))))$

* (2) $\forall n(2*n+1 \text{ is odd} \leftrightarrow(\forall m\, (m>n \rightarrow(2*m+1 \text{ is odd} \rightarrow \Phi))))$ (1)

* (3) $(\text{True} \leftrightarrow (\text{True} \rightarrow \Phi))$ (2) from elementary arithmetic

*(4) Φ (3)

(5) If $\exists\alpha\forall n\, (\alpha(n) \leftrightarrow (\forall m\, (m>n \rightarrow(\alpha(m) \rightarrow \Phi))))$ then Φ (4)

Q.E.D.

So I verified booth Cook's claims, and it is important to note that I used only elementary arithmetic theorems, without using T schema.