

Tarski against quotation functions

Let  $\#(x)$  be a “quotation function” of the sort Tarski mentions on p. 161. Thus ‘ $\#(\text{snow is white})$ ’ is an expression that denotes the sentence ‘snow is white’:  $\#(\text{snow is white}) = \text{‘snow is white’}$ . Assume that quantification into the quotation functor is explained substitutionally: ‘ $\forall p \dots \#[p] \dots$ ’ is true iff for every sentence  $p$  belonging to the substitution class ‘ $\dots \#[p] \dots$ ’ is true.

Then Tarski claims that a contradiction can be derived from:

- (a) ‘ $\forall p (c = \#[p] \rightarrow \sim p)$ ’ =  $c$   
and
- (b)  $\forall p \forall q (\#[p] = \#[q] \rightarrow p \text{ iff } q)$

The argument presumably goes as follows:

Assume  $c$  is true:

- (1)  $\forall p (c = \#[p] \rightarrow \sim p)$
- (2)  $c = \#[\forall p (c = \#[p] \rightarrow \sim p)] \rightarrow \sim \forall p (c = \#[p] \rightarrow \sim p)$  (1), Univ. Inst.
- (3)  $c = \#[\forall p (c = \#[p] \rightarrow \sim p)]$  (a)
- (4)  $\sim \forall p (c = \#[p] \rightarrow \sim p)$  (2), (3)
- So  $c$  is false:
- (5)  $\sim \forall p (c = \#[p] \rightarrow \sim p)$
- (6)  $\exists p \sim (c = \#[p] \rightarrow \sim p)$  (5)
- (7)  $\sim (c = \#[q] \rightarrow \sim q)$  (6),  $q$  Exist. Inst.<sup>1</sup>
- (8)  $c = \#[q] \ \& \ q$  (7)
- (9)  $\#[\forall p (c = \#[p] \rightarrow \sim p)] = \#[q] \ \& \ q$  (3), (8)
- (9.1)  $\#[\forall p (c = \#[p] \rightarrow \sim p)] = \#[q]$  (9)
- (10)  $\#[\forall p (c = \#[p] \rightarrow \sim p)] = \#[q] \rightarrow \forall p (c = \#[p] \rightarrow \sim p)$  iff  $q$  (b), Univ. Inst.
- (11)  $\forall p (c = \#[p] \rightarrow \sim p)$  (9.1), (10)

So  $c$  is true: contradiction.

The problem is that steps (2) and (10) are fallacious: the allowable substituends for the variable ‘ $p$ ’ should be members of the substitution class, and these will not be arbitrary sentences of the language (and certainly not sentences containing the substitutional quantifier itself). See Marcus, “Quantification and Ontology”, 83-4 (in *Modalities*); Kripke, “Is There a Problem about Substitutional Quantification?”, 367-8 (in Evans and McDowell, *Truth and Meaning*); Soames, appendix to ch. 3 of *Understanding Truth*.

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<sup>1</sup> ‘ $q$ ’ is a sentence letter (‘ $p$ ’ and ‘ $q$ ’ are sentential variables).